Optimization design and thinking of new knowledge of primary school mathematics under the perspective of high-order thinking cultivation: Take the surface area of the cylinder as an example

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ABSTRACT
The 20th report clearly emphasizes the importance of training innovative talents, and the cultivation of higher-order thinking has been paid attention to. Micro-lesson is a new teaching method to cultivate and develop students' high-level thinking. Taking cylindrical surface micro-lesson design as an example, this paper, starting from practice, compares the micro-lesson design and fragment record before and after optimization, and puts forward suggestions for optimization design: Pay attention to the construction of knowledge system and develop general thinking; Pay attention to summary reflection, form critical thinking; Pay attention to situation creation and develop innovative thinking; Focus on problem chain design and develop logical thinking. In order to provide reference for primary school mathematics classroom micro-lesson design.

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INTRODUCTION
The Central Committee of the Communist Party of China and The State Council issued the Opinions on Deepening Education and Teaching Reform and Comprehensively Improving the Quality of Compulsory Education, which requires strengthening the role of the main position of the classroom, using modern technological means to develop new types of teaching that can inspire students' thinking and enhance their higher-order thinking ability, and improve the quality of classroom teaching (Nianzhou, 2019). The 20th report also clearly emphasizes the importance of training innovative talents Under the (Yongxin, 2022). A primary school teacher in Shanghai conducted a survey on the high-order thinking ability of senior primary school students, and the results showed that the mathematical high-order thinking ability of primary school students is generally low, and the development is unbalanced (Qiong, 2022). As the basic stage of students' high-order thinking training, the integration of micro-class and mathematics teaching does not pay
attention to the development of pupils’ thinking, so it is urgent to cultivate their high-order thinking ability in senior grades. Aiming at the research point of how to develop students' higher-order thinking in knowledge micro-lessons, this paper attempts to put forward relevant optimization suggestions through comparative analysis of the design before and after micro-lessons, so as to provide references for the design of micro-lessons in primary school mathematics classrooms.

**Mathematical higher-order thinking**

Generally speaking, higher-order thinking refers to the mental activity or cognitive ability that occurs at higher cognitive levels (Jun, Zhihong & Nan, 2021). It is still difficult to give an accurate definition of mathematical higher-order thinking. There are many concepts about higher-order thinking ability in foreign countries, such as "analysis, synthesis and evaluation", "introspective thinking", "Triarchic theory of intelligence" and "three-dimensional structure model of intelligence of divergent thinking" (Jinmei, 2022). Domestic research on higher order thinking started relatively late. Zhixian (2004) emphasizes that higher-order thinking is a kind of cognitive activity occurring at a higher level, which includes two typical thinking abilities, critical thinking and creative thinking, plus problem decision-making and problem solving. Youxiang (2017) believes that it is a rational activity that makes reasonable use of abstract thinking structure in the process of interaction between human brain and mathematical objects such as spatial form, quantitative relation and structural relation. Liguang and Qiong (2019) believe that a comprehensive thinking process which experiences connection and transformation, abstraction and expansion, criticism and monitoring is the higher-order thinking. According to Ping (2021), higher-order thinking mainly consists of logical reasoning, problem solving, generalization of knowledge system, critical thinking and creative thinking. Based on the concept definition of mathematical higher-order thinking given by domestic scholars, the basic components of higher-order thinking in primary school mathematics include at least four elements: general thinking, critical thinking, innovative thinking and logical thinking (Ping, 2021).

**CONCEPT OVERVIEW**

**Micro lesson**

As it runs through the whole process of the cultivation of core qualities, it is usually an important goal of education (Ningzhong, 2017). In China, Tiesheng (2011) from the Education Bureau of Foshan City, Guangdong Province, first proposed micro-lesson, which is an organic combination of learning resources involved in the learning activities designed by teachers for a specific knowledge point or knowledge link. Jiao Jianli, on the other hand, believes that micro-lessons are teaching videos presented in the form of short online videos for students to learn and use for the purpose of explaining a specific knowledge (JJianli, 2013). Jiahou (2013) believes that micro-lesson is a small course, which consists of specific learning goals and its content is relatively brief and concentrated (Jiahou, 2013). Based on the above definition of micro-lessons, newly taught knowledge micro-lessons enable teachers to analyze the key and difficult points of new knowledge points according to the teaching materials and curriculum standards, so that students can understand a new knowledge point in a short time. This kind of micro-lesson can reduce the abstract level of mathematics, stimulate students' learning interest, meet students' personalized learning, mobile learning and fragmented learning, and develop primary school students' core literacy and thinking ability (Qiaohua, 2022).

**Basic analysis of the micro-lesson content**
The surface area of a cylinder is an important knowledge point in the chapter of Cylinders and Cones, which is the basis for learning other geometric knowledge. Before this, students have learned the surface area and volume of cubes and cuboids (Junyu, Zhilan & Jianlan, 2020). Moreover, students have understood the basic characteristics of the cylinder and explored the side expansion diagram of the cylinder by hand, and have the basis for studying the surface area of the cylinder. The teaching of this knowledge can cultivate students' abstract generalization ability, thinking ability and establish spatial concept, which has a positive impact on the development of students' higher-order thinking. Because the spatial imagination of primary school students is not rich enough, and the abstract degree of the calculation of the side area of the cylinder is high, the difficulty of this micro lesson is set as: analogy the process of exploring the surface area of the cube and cuboid, and deduce the calculation formula of the side area and surface area of the cylinder in combination with the cylindrical expansion map learned in the last hour (Hongzhen, Chunna & Xiaotang, 2020). In traditional teaching, it is difficult to show the development process of columns intuitively. Ignoring the relationship between columns, cubes and cuboids, separating them, over-instilling knowledge and belittling students' independent learning, not only hinders the establishment of students' spatial concepts, but also impedes the construction of students' complete knowledge system and makes it difficult to develop holistic, creative, logical and other higher-order thinking (Yiting, Yifan & Jianlan, 2022).

Design or comparative analysis of micro-lesson optimization
Optimization before micro lesson (hereinafter referred to as the "original") of the basic link of the design: review introduction-warm up the old and introduce the new-deep analysis-problem solving-inductive summary-consolidation after class; optimized micro lesson (hereinafter referred to as "optimization") design: story introduction-analogy inquiry,explore new knowledge-application of knowledge-summary reflection-after-class inquiry(Caibin, 2021). The design or transcript of each of these links is compared and analyzed below.

Comparative analysis of the original "Review import" and the optimized "story introduction" design. As shown in Figure 1, The original version is introduced from the perspective of old knowledge. By reviewing the characteristics of the cylinder, it raises the question: how to calculate the surface area of the cylinder? The optimized version first set up a story situation: the class organized spring outing, a few students to discuss the problem of painting cylindrical wood, which leads to the topic of this lesson: how to calculate the surface area of a cylinder. Compared with the original version, the optimized version combines students 'life to create an interesting situation, which fits the learning situation, and leads to a more natural question, which can mobilize the enthusiasm of students in learning, activate students' thinking, and stimulate students' interest in learning (Dongke & Yue, 2020). Whether students' thirst for knowledge is stimulated or not determines the degree of students' investment in the learning process, and then affects the development of high-order thinking.
Comparative analysis of the design of the original version of "warming the old and introducing the new knowledge" and the optimized version of "analogy exploration"

As shown in Figure 2, both the original version and the optimized version adopt a design method that is similar to the exploration process of cube and cuboid surface area. When reviewing the exploration process of cube and cuboid surface area, the original version only gives a suggestion from space problem to plane problem, which has a high degree of abstraction, does not conform to the cognitive development rules of primary school students, and fails to develop students' general thinking well. The optimized version breaks down the whole exploration process, triggers students' memories in the form of a chain of questions, converts the problem of finding the surface area of cube and cuboid into finding the area and problem of the expanded graph of cube and cuboid, and leads students to summarize the steps of exploring the surface area of three-dimensional graph, linking cube, cuboid and cylinder. Through the thinking process from concrete to abstract, the degree of abstraction is reduced, and it is easy for students to understand. At the same time, it can well develop students' integrity and logic, two kinds of higher-order thinking ability (Jian-lan, 2015).

Comparative analysis of the original "Profound Analysis" record and the optimized version of "Exploring New Knowledge" record

As shown in Figure 3, in the original version, students are asked to experience the process of cylinder development in the form of group cooperation, so as to get the cylinder development diagram. The optimized version is with the help of Hao Jun dynamic technology, to show students the process of cylinder development directly, with information technology instead of students' direct experience. The original version and the optimized version have a similar process of exploring the surface area after drawing the expansion drawing, but the optimized version adds one more question than the original version, that is, what is the difference between the surface area and the side area, as shown in Figure 4.
Teacher: Please take out the cylindrical object prepared before class, take the group as a unit, cut along the height of the cylinder, observe which parts of the cylinder is composed of.

Student: It consists of two circles and a cuboid.

Teacher: So, what is the surface area of the cylinder?

Student: The surface area of the cylinder is equal to the side area plus two base areas.

Teacher: Great! So let's think about two questions with this expansion of a cylinder, can you calculate the side area of a cylinder? What about the base area?

Student: The bottom is a circle, so the area of the two bottom is the area of the two circles. The side of the cylinder is a surface, and when you expand the side you get a rectangle, so the side area of the cylinder is equal to the area of the rectangle.

Teacher: We know that the formula for the area of a rectangle is length times width. What is the length and width of a rectangle here?

Student: Here the length of the rectangle is equal to the circumference of the bottom of the cylinder, and the width of the rectangle is equal to the height of the cylinder.

Teacher: So we only need to know the circumference and height of the base of the cylinder to find the side area of the cylinder.

Teacher: If h is used to represent the height of a cylinder, can you use letters to show the formula for calculating the area of the side of the cylinder?

Student: If we know that the circumference of the bottom side of the cylinder is big C, then we can directly calculate the area of the side, denoted by the letter. If we know that the diameter of the base is little d, since the circumference of the base C is equal to, the area of the side is denoted by the letter. If the base radius is known to be small r, the side area can be denoted by the letter since the diameter d is equal to the product of twice the radius 2r.

[Optimized version snippet recording]
Teacher: We have learned before, along the height of the cylinder can be cut to get the cylinder development diagram, let's review its development process with the help of Hao Jun dynamic technology.

Teacher: After watching the dynamic development process of the cylinder, observing the development diagram, what can you find?

Student: It can be found that the expanded diagram of a cylinder includes two bases and one side, so the surface area of the cylinder is equal to the area of the side of the cylinder plus the area of the two bases.

Teacher: What a careful observation! Think about two questions using this expansion diagram, can you calculate the side area of a cylinder? What about the base area? Let's look at the base area first.

Student: The area of the two bottom sides of a cylinder, in fact, is to find the area of two circles, if you use \( r \) to represent the radius of the bottom, the area of the circle is, then the letter is.

Teacher: Then look at the side area of the cylinder. When the side is expanded, what figure is it?

Student: It's a rectangle.

Teacher: How to find the area of the rectangle?

Student: Length times width.

Teacher: What about the length and width of the rectangle here?

Student: The length is equal to the circumference of the base of the cylinder, and the width is equal to the height of the cylinder.

Teacher: Very good! Similarly, if the height of a cylinder is expressed in terms of \( h \), can you write out a formula for calculating the area of the side of the cylinder?

Student: If we know that the circumference of the bottom side of the cylinder is big \( C \), then we can directly calculate the area of the side, denoted by the letter \( Ch \). If we know that the diameter of the base is little \( d \), since the circumference of the base \( C \) is equal to, the area of the side is denoted by the letter. If the base radius is known to be small \( r \), the side area can be denoted by the lettersince the diameter \( d \) is equal to the product of twice the radius \( 2r \).

Teacher: According to the above exploration, how can you find the difference between the surface area and the side area of the cylinder?

Student: the side area is a part of the surface area, and the surface area also includes two bottom areas.

Analysis: Both the original version and the optimized version use the form of the question chain to explore the new knowledge, but compared with the original version, the question chain design of the optimized version is more specific and reasonable. At the same time, the optimized version can be reviewed using dynamic technology based on the fact that the students have already learned about cylinder development diagrams, while the original version uses hands-on exploration. Although cooperative inquiry plays an important role in giving play to students' dominant position and can develop students' high-order thinking at the same time, it ignores the learning situation here, which is a meaningless repetition. Reasonable problem chain design can promote students to think continuously and effectively, guide students to gradually deepen the original knowledge structure, and effectively ensure the relevance and construction of mathematical knowledge, so as to develop high-order thinking(Ma Shufeng & Yang Xiangdong.(2022)). The micro-course design of the optimized version is developed with a ladder chain of questions, which guides
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students to find problems from the mathematical situation of the expansion graph and abstractions the graph relationship from the spatial form, which can well train students' mathematical logical thinking and abstract thinking. And the optimized version of the dynamic technology to assist teaching, not only can make up for the underachiever in mathematical intuition and imagination ability of part of the shortage, but also can stimulate the mathematics intuition and imagination potential of top students. Finally, the optimization version also adds the question of comparing the surface area and side area of the cylinder, which is more conducive to students to clarify the substantive connection between knowledge and construct a more complete mathematical knowledge network structure, which is an effective way to develop mathematical logical thinking.

Comparative analysis of the original "Problem solving" design and the optimized version of "Learning to use" design

As shown in Figure 5, the questions of the original version and the optimized version are both related to the calculation of the surface area of the cylinder. The difference is that the questions of the optimized version are the concrete performance of the story in the "story introduction" section. The topic gives the specific numerical, and specific situation, so that students can flexibly and comprehensively apply the mathematical knowledge and methods learned in this micro-lesson to solve practical problems. Paying attention to the connection and comprehensive application of mathematics and life is the manifestation of the development of students' innovative thinking in mathematics. While consolidating students' knowledge, students can improve their ability to observe the world with mathematical vision, think about the world with mathematical thinking and express the real world with mathematical language (Liguang & Qiong, 2019).

Comparative analysis of the original "induction summary" and the optimized version of "summary reflection" design

Both versions are used in the form of flow chart to sort out the knowledge learning process of this micro-lesson. The original version emphasizes the acquisition and summary of students' mathematical knowledge, while the optimized version emphasizes the reflection and evaluation of students' mathematical thinking and thinking methods while focusing on students' harvest, so as to better develop students' mathematical critical thinking and cultivate their higher-order thinking ability.

Comparative analysis of the original "after-class consolidation" and the optimized "after-class inquiry" design

In the original version of the "after-class consolidation" section, students are left with homework exercises in the textbook, while the optimized version of the micro-class sets an open topic for students to explore. The optimized version of the "Learning to Use" section sets up the situation,
"What are some life scenarios related to surface area calculation of a cylinder? Can you illustrate and calculate?". In view of this problem, students can make different explanations according to their own life experience, such as "give cylindrical pen holder paste colored paper, seek the area of colored paper", "use iron sheet to make cylindrical iron bucket, need the area of iron sheet" and so on. In this process, students think from different angles, get different results, and their innovative thinking is improved to a certain extent.

**Thinking on the design of new knowledge teaching micro-course from the perspective of higher order thinking training in mathematics**

Through the comparative analysis before and after the micro-lesson optimization of Surface Area of a Cylinder, the focus of micro-lesson design optimization is to optimize the situation design and problem chain setting according to several basic components of the advanced thinking of primary school mathematics, so as to develop the advanced thinking of primary school mathematics. To this end, I propose the following four thoughts:

**Pay attention to the construction of knowledge system and develop general thinking**

Mathematical knowledge system is the foundation of mathematical subject, which includes all branches, concepts, theorems, formulas and so on. Students need to establish a complete mathematical knowledge system through systematic learning and mastering, so as to better understand and apply mathematical knowledge (Miao, Li & Jianlan, 2022). In the process of micro-lesson design, we should pay attention to the construction of students' complete knowledge system, help students to integrate the old and the new knowledge together, build a dynamic cognitive model, and develop students' ability of knowledge transfer and problem-solving ability, so as to develop general thinking. In order to develop students' general thinking through the construction of mathematical knowledge system, we can start from the following aspects: First, we can provide students with diversified examples. In the micro-course design, students can set multiple examples, such as promotion problems, applied problems, etc., from different angles, and gradually form a general thinking by drawing inferences from other cases. Secondly, in the reflection summary of the micro-class or the after-class consolidation link, students can be encouraged to self-summarize the content of this section. Let the students summarize the content by themselves, and enumerate the rules and characteristics of the knowledge, which is an effective way to develop the general thinking through the construction of the knowledge system. Thirdly, you can also set up a multi-solution classroom consolidation topic, to present students with different solutions, let students analyze, compare and identify different solutions, so as to form a general thinking. Finally, in the process of micro-lesson design, mind mapping can also be used to classify different knowledge points, so that students can more intuitively understand the level and connection of mathematical knowledge, and cultivate students' ability of general thinking.

**Pay attention to summary reflection and form critical thinking**

In micro-lesson design, summarizing and reflecting can help students develop critical thinking in mathematics. Specifically, the following measures can be taken: Guide students to summarize the thought methods and knowledge skills learned in this lesson, and let students think about how to apply these thought methods and knowledge skills to solve practical problems in life; Remind students to think about whether the concepts, theorems and formulas in this lesson are reasonable and correct, let students try to put forward their own doubts and solutions; Encourage students to think critically about the application feasibility and limitations of mathematics in combination with...
practical problems, so that students can understand the limitations of mathematics and the complexity of practical applications; Students are encouraged to think about the cross-application of mathematics with other disciplines and fields, and to creatively apply mathematical knowledge to solve complex problems. The above ways of reflection can help students develop critical thinking, cultivate their curiosity and thirst for knowledge, stimulate their interest in learning, and implement the trinity of optimization mathematics teaching concept of "desire for fish and fish"(Jian-lan, 2015). At the same time, it is also an effective way to promote the comprehensive development of students' mathematics quality.

Pay attention to situation creation and develop innovative thinking
Thinking occurs in a certain situation, and the creation of the situation is a necessary condition for the occurrence of thinking, which is also a necessary condition for the occurrence of higher-order thinking (Ping, 2021). Pupils' innovative thinking can be reflected in different interpretations of a problem. Therefore, open life situations can be added to micro-class design to combine life and mathematics, which is conducive to students' understanding and application of knowledge. Students' various explanations can also help students develop innovative thinking. At the same time, challenging questions can be designed appropriately. Challenging questions can make students explore and try to solve problems actively, and challenging questions can help students find new ideas and new methods in the process of exploration. In addition, interdisciplinary learning situations can also be created, involving different disciplinary knowledge, skills and thinking methods, such as mathematics, physics, computer, etc., in which students explore problems and propose innovative solutions, thus developing innovative thinking. Finally, it can also create contradictory situations to cause cognitive conflicts among students, so that they can enter deep thinking and break the thinking pattern, thus generating new ways of thinking and solutions.

Pay attention to the problem chain design and develop logical thinking
The emphasis on problem chain design and the development of logical thinking problems are known as the starting point of thinking. In mathematics, a highly abstract subject, the development of thinking activities cannot be separated from the strong support of the carrier of problems (Jun, Xiaoni & Jianhua, 2021). The progressive problem chain is closely related to the logic and structure of mathematics and fits each other. The progressive question chain connects the preceding and following questions in the form of series, and guides students to move forward through layers of progressive logical sequence. The use of progressive chain of questions can help students to clarify the internal logic between knowledge, so as to develop logical thinking. In the process of learning, students experience a gradual process from the simple to the deep, at the same time, the students' thinking gradually from low to high level development. Therefore, when designing the question chain, the following problems should be paid attention to: The difficulty of the question design should be moderate in accordance with the law of students' cognitive development. Too simple questions are difficult to stimulate the cognitive conflict of students, and can not effectively cultivate the depth of students' thinking. However, if the problem is too difficult, students will not be able to start and their thinking will be stalled. Therefore, when designing question chain, teachers should consider the framework of the whole knowledge system, systematize knowledge, level questions, and make thinking logical. Only when the question chain is logical and hierarchical, can students be guided to rational reasoning and analysis, so as to develop their high-level thinking.
This study focuses on the combination of mathematical higher-order thinking and micro-lesson. The advantages of micro-lesson can help develop students' higher-order thinking. Therefore, four suggestions are put forward to develop students' higher-order thinking in micro-lesson design, and the surface area of the cylinder is optimized as an example. Later educational practice can be carried out to verify whether the optimize the micro-lesson before and optimized micro-lesson change students' high-order thinking level. However, because the change of thinking needs a certain amount of time, it is necessary to use micro-lessons, rather than only a few micro-lessons. I hope this study can provide some inspiration for scholars to study high-level thinking and the application of micro-courses.

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