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Mathematical Concept Teaching Design based on APOS Theory in the Context of Mathematical History: A Plane Rectangular Coordinate System as an Example

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ABSTRACT

Mathematics learning aims to train students to observe, think, and express the real world with mathematical vision. However, in the traditional static concept teaching mode, teachers tend to use the concept formation or expository method, so most students cannot understand abstract concepts in class but have to do exercises to consolidate their understanding of concepts, which does not meet the requirements of cultivating talents with core mathematical literacy. Literature research method: To understand the current research status of APOS theory on mathematics concept teaching at home and abroad, this study consulted relevant books, journals, papers, and network resources at home and abroad. This paper discusses it based on the history of mathematics and provides practical guidance for teachers on designing student-centered teaching activities based on APOS theory. Based on the teaching design under the APOS theory, problem-solving is used to gradually guide students to abstract mathematical concepts and learn to think about the real world with mathematical thinking. Teachers will also diagnose students' learning stages through problem-solving situations and constantly debug math classes suitable for students. In guiding students to think, explore and interact, teachers create cognitive conflicts many times, encourage students to question questions, and give full play to students' subjectivity.

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INTRODUCTION

Mathematical concept is not only the foundation of mathematical knowledge system, but also the key for students to master mathematical knowledge and form basic skills. But its highly abstract nature and broad generalities make it difficult for some students to understand the nature of these concepts. The current teaching of mathematical concepts has problems such as emphasizing memory, undervaluing process, undervaluing practice and undervaluing essence (Zhao, 2021). Although the Compulsory Education Mathematics Curriculum Standards (2022 edition) (Ministry

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of Education of the People's Republic of China, 2022; Tang et al., 2023) pointed out that students' learning should be an active process, and teaching activities should focus on guiding students to find and raise problems, analyze and solve problems with various methods. However, it is still difficult for teachers to exert students' learning initiative effectively in the teaching of mathematical concepts, which requires teachers to design teaching activities that not only stimulate students' interest, but also promote students' in-depth understanding of mathematical concepts. At present, APOS theory is a good way to teach mathematical concepts in the eyes of Chinese scholars, but the effective implementation of APOS theory requires teachers to have a more professional and in-depth understanding of the taught knowledge, which also becomes one of the reasons limiting the effective application of APOS theory in the classroom (Cai, 2019). The history of mathematics not only provides teachers with an objective source of professional in-depth understanding of the formation of mathematical concepts, but also adds interest to the mathematics classroom and attracts students' attention. Therefore, this paper takes the teaching design of "plane Cartesian coordinate system" as an example to discuss the teaching strategy based on APOS theory mathematics concept from the perspective of mathematics history.

Mathematical concept is not only the foundation of mathematical knowledge system, but also the key for students to master mathematical knowledge and form basic skills. But its highly abstract nature and broad generalities make it difficult for some students to understand the nature of these concepts. The current teaching of mathematical concepts has problems such as emphasizing memory, undervaluing process, undervaluing practice and undervaluing essence (Zhao 2021). Although the Compulsory Education Mathematics Curriculum Standards (2022) (Ministry of Education of the People's Republic of China, 2022; Tang et al., 2023) pointed out that students' learning should be an active process, and teaching activities should focus on guiding students to find and raise problems, analyze and solve problems with various methods. The purpose of mathematics learning is to train students to learn to observe, think and express the real world with mathematical vision. However, in the traditional static concept teaching mode, teachers tend to use the concept formation or expository method, so that most students cannot understand abstract concepts in class, but have to do exercises to consolidate the understanding of concepts, which obviously does not meet the requirements of cultivating talents with core mathematical literacy.

APOS theory is composed of action, process, object and schema. If the individual is guided through several stages of thinking, such as operation, process and object, the individual can generally organize them into schemas on the basis of construction and reflection so as to clarify the problem situation and solve the problem smoothly (Ed Dubinsky&Michael A 1993). Moreover, the psychological mechanism of mathematics learning is a kind of ring structure, in which operations, processes and objects are finally combined with the original schema to form a new schema, or they can directly act on the object and react on the process to form a new schema(Xiaodan Ma 2020). Eventually, schemata will either become stable structures or develop dynamic mechanisms that can assimilate other schemata, and a comprehensive psychological schema will be formed between schemata and schemata. Since the attention of Chinese scholars in the 1990s, APOS theory has been used by scholars and teachers as a theoretical basis to guide concept teaching. Scholar Lianquan Qiao (Lianquan Qiao 2003) analyzed the changes in students' performance in learning Lagrange's theorem, normal subgroups, and business groups, and demonstrated that APOS theory can help teachers learn some teaching strategies to guide students to learn complex or abstract mathematical concepts. Chenghua (Chenghua 2010) thought about the teaching of mathematical concepts based on the APOS theory, designed the stages of operation, process, object and schema as a whole, and used operation as the guide, allowing students to reflect

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in the process and experience each stage of concept formation completely. Emphasis is placed on the process of concept formation, as well as on the diversity of teaching, so that students can reach different levels of understanding. Huangbin (2023) also discussed that under the guidance of APOS theory, teachers guide students through the four stages of thinking, which not only enables students to have a full understanding of the concept of function, but also cultivates students' core literacy of mathematics.

LITERATURE REVIEW

APOS Theoretical Definition

APOS theory, proposed by Dubinsky and others, aims to guide the teaching practice of mathematical concepts. APOS theory consists of action, process, object and schema, in which the operation stage is students' initial contact and understanding of mathematical concepts, perceiving and experiencing mathematical concepts through actual operations or activities. On the basis of operation, the process stage synthesizes the operation of concepts into a conceptual process, which helps students transition from concrete operation to abstract concept understanding. The object stage is a further development of the process stage, where students begin to be able to independently work with concepts, analyze them and apply them. In the final schema stage, students are able to integrate the concepts they have learned into the broader mathematical knowledge system to form a comprehensive mental schema (Li Zhijia, 2019). If led the individual through the thinking process of operation, and the object after several stages, such as individual generally can be constructed and put them on the basis of the reflection of schema to clarify problem scenarios, fix the problem (Ed Dubinsky & McDonald 2001).

APOS Theory in China

Since the attention of Chinese scholars in the 1990s, APOS theory has been used by scholars and teachers as a theoretical basis to guide concept teaching. Scholar Qiao (Lianquanqiao 2003) proved that APOS theory can help teachers learn some teaching strategies to guide students to learn complex or abstract mathematical concepts by analyzing students' performance changes in learning Lagrange's theorem, normal subgroups and business groups. Cheng Hua(Cheng Hua 2010) thought about the teaching of mathematical concepts based on the APOS theory, designed the operation, process, object and concept stage as a whole, and used the operation as the guide, so that students could reflect in the process and experience the various stages of concept formation completely. Emphasis is placed on the process of concept formation, as well as on the diversity of teaching, so that students can reach different levels of understanding.Huangbin(Huangbin 2023) also discussed that under the guidance of APOS theory, teachers guide students through the four stages of thinking, which not only enables students to have a full understanding of the concept of function, but also cultivates students' core literacy of mathematics. To sum up, APOS theory is a good way to teach math concepts in the eyes of Chinese scholars, which not only meets the requirements of national curriculum standards for math classes, but also helps math concepts become interesting and easy to understand.

Research Question and Research Purpose

When helping students to feel the background of concepts based on APOS theory activities, teachers often teach according to the logical arrangement in the textbook, so that students only see the logical relationship between knowledge, but do not understand the context of knowledge and the connection with the real world. In order to stimulate students' interest in learning, teachers often

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design teaching activities by using short stories in the history of mathematics, which is the objective source of the formation process of mathematical concepts and the focus of the curriculum concept in the latest curriculum standards. Therefore, how to integrate mathematical historical materials well into the teaching of mathematical concepts based on APOS theory is the focus of this paper. This paper takes the concept lesson of "Plane rectangular coordinate system" as an example to inspire teachers how to design interesting but profound teaching activities based on APOS theory in the teaching of mathematical concept lesson with the help of the history of mathematics to help students grasp the essence of mathematics.

METHOD

Literature research method: In order to understand the current research status of APOS theory on mathematics concept teaching at home and abroad, this study consulted relevant books, journals, papers and network resources at home and abroad; This paper discusses it based on the history of mathematics and provides effective guidance for teachers on how to design student-centered teaching activities based on APOS theory.

The case study method takes the concept teaching design of "plane rectangular coordinate system" as a case, explores the source of material selection with the history of mathematics as the background, and designs the concept teaching process with APOS theory. The teaching design in this paper focuses on the process of guiding students to explore independently and communicate with each other. Compared with the use of "number line" in the textbook, it directly proposes "Can you determine the position of plane points by using similar methods?" The problem is more conducive to students' thinking, more conducive to the cultivation of students' core quality of geometric intuition and application consciousness. Based on the above discussion, this paper summarizes the strategy of designing mathematics concept teaching activities based on APOS theory, hoping to enlighten teachers' activity design of mathematics concept class

RESULTS

Teaching Designing of Plane Rectangular Coordinate System Method

The beginning of the plane cartesian coordinate signifies the birth of plane geometry, and its appearance can enable people to solve the problem that cannot be solved by the rule drawing. The beginning of the plane cartesian coordinate system is not the form of "double axis", but from the "single axis", some scholars gradually introduce the coordinates of "negative range" and eventually evolve into "double axis". Seventh grade students have already mastered the concept of a "number line," after learning that real numbers correspond one-to-one to points on the number line. How to guide students to transition from "one-dimensional" schema to "two-dimensional" schema is the focus and difficulty of this concept lesson.

Operation and Process

Let students realize that "real numbers in a number line are not enough to describe the position of points in the plane", combine APOS theory and mathematical history knowledge to create a problem situation, so that students initial contact and understanding.Next, the question chain is used to guide students to establish a second "number line". The answer to each question is an "operation" in APOS theory. Guiding students to form the corresponding relationship between points on the plane and two real numbers through repeated "operations" is the "process" in APOS theory

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Question 1: Do students know the mathematician Descartes? In Descartes's day, both geometry and algebra flourished, but those who majored in geometry thought that algebra was too monotonous, and those who majored in algebra thought that geometry was not rigorous. Descartes thought, Can you relate geometry to algebra? Descartes found that the most basic element in geometry is "point", and the most basic element in algebra is "real number". Can we use real numbers to represent points? Students, can you help him with what you've already learned?

[Design intention]: Creating the problem situation through mathematical historical materials can not only let students understand the cause of the plane rectangular coordinate system, but also guide students to establish the connection between the "real numbers and number lines" schematically. Through problem solving, students are guided to think step by step.

Question 2: On the number line it is easy to answer Descartes's question because of the one-to-one correspondence between the real numbers and the number line, but Descartes thought that the number line was just a line, which was not enough for the plane, as shown in the figure.When there is A point A that is 4 units above the real number 3, can it be represented just by the number line we know? How many quantities do I need to represent point A? The other points B,C,D in the plane can you also use the same method to represent the points in the plane as numbers?

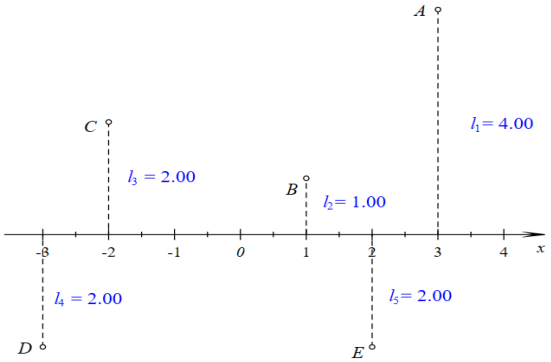

Figure 1. Coordinate System

[Design intention]: To induce students to realize that a point in the plane cannot be represented by one real number, and only two real numbers can be used to represent a point in the plane, so that students have cognitive conflicts, guide students to think independently, explore independently, cultivate students' learning habits of independent thinking, and encourage students to answer questions. At this time, the answer must be a hundred flowers. The process of establishing A relationship between the position of point A in the plane and the real numbers is an "operation", as is the case with other points. Through repeated "operations", students are guided to form a "process" model about the correspondence between points on the plane and two real numbers.

Object

After the students have a preliminary understanding of the "plane rectangular coordinate system", the teacher helps the students to take the "plane rectangular coordinate system" as the "object", gradually confirm its positive direction, number line position, and use ordered real number pairs to represent the position of points in the plane.

Question 3: In the figure, point B is 1 away from the number line, which confirms the unit length in this plane, and the four points A,B,C,D are either on the number line or below the number line, if we set up to +, then we set down to -, what do you think of this?

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Question 4: How to determine the origin of another number line? Can you show me where the other number line is? Can we really simplify the expression by determining another number line? How can you express A, B, C, D? Through group discussion, say your group's answer.

Question 5: There is ambiguity in the answer of some students, some people say that 1 2 and 2, some people say that 2 and 1 2, then how to represent the point E? In order to distinguish the two points, we stipulate that the horizontal coordinates are in front, and the ordinate coordinates are separated by commas in parentheses. For example, the point C is represented by (-2,2), and the point A is represented by (3,4), so that C and E can be distinguished. This is an ordered number pair.

[Design intention] Cognitive conflict is constantly used to help students learn to apply "object", so as to improve the definition of "plane rectangular coordinate system". Through discussions among students, students can give full play to their initiative and train students to express the world scientifically and rigorously in mathematical language.

Schema

The concept of building a "plane rectangular coordinate system" is the "diagram" formed in this lecture. Question 6: Can every point in the plane now be represented by a coordinate? Do we in turn list a coordinate and determine that point in the plane cartesian coordinate system? So points in the plane are one-to-one correspondences to _____?

[Design intention] Some exercises will be designed here to help students master the conversion between plane points and coordinates. In APOS theory, problem solving can be used to diagnose the learning stage of students.

[Class summary] In reality, there are other ways to express the position of the point, such as the direction Angle used to express the position of the object in navigation, the real number and Angle are used to express the position of the point, and on the earth, two angles are used: longitude and latitude to express the position of the point. "Ordered real pairs" in mathematics and "longitude and dimension" in geography become different ways to represent the position of a point in a plane, so we can see that two quantities are needed to represent the position of a point in two-dimensional space, not necessarily both real numbers.

[Design intention] Using the way of expressing the position of points that is different from the "plane rectangular coordinate system" encountered in navigation and geography, it inspires students to "express points in the plane is actually using two quantities", and helps students realize the transfer of "illustration". In this process, it helps students to exercise the ability of combining number and form, and the core qualities such as geometric intuition and application consciousness have been cultivated

DISCUSSION AND CONCLUDING REMARKS

The perspective of mathematics history is based on APOS theory mathematics concept teaching strategy.

The Teaching Situation is Designed According to the Formation Process of Concept in History

Using the history of mathematics to create a problem situation can make mathematics teaching more vivid and interesting, help to dilute the abstractness of mathematics, and inject new vitality into the dull classroom (Su, 2007). When creating problem situations, teachers use the history of mathematics to guide students to feel the source of knowledge, which is not only in line with the

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historical development, but also in line with the needs of students' cognitive development, so that students can experience the emergence and development of the plane rectangular coordinate system and gradually build psychological structure, which can also enables the professional quality of teachers to develop (Putri et al., 2023).

Design Teaching Activities Based on Students' Learning Situation

Mathematical concepts in the long history of the formation is not an overnight, such as the "plane cartesian coordinate system", Descartes originally used the coordinate system, the Angle between the two number lines is not 90°, for seventh grade students too abstract, it is difficult to use. The focus of this lesson is to help students transition from "one-dimensional" to "two-dimensional", so this lesson sets teaching activities on the basis of "using two real numbers to represent the position of the point in the plane". When designing teaching activities, teachers should not be divorced from students' learning situation, but focus on the presentation of the essence of knowledge, so that students can feel the historical charm of the formation process of mathematical concepts in the classroom with limited ability.

By Constantly Creating Cognitive Conflict, Students are Constantly Guided to Think Independently

Based on the teaching design under the APOS theory, problem solving is used to gradually guide students to abstract mathematical concepts and learn to think about the real world with mathematical thinking. Teachers will also diagnose the learning stage of students through the problem-solving situation, and constantly debug the math class suitable for students. In the process of guiding students to think, explore and interact, teachers create cognitive conflicts for many times, encourage students to question questions, and give full play to students' subjectivity. In this case, students will have a deeper understanding of the "bridge between geometry and algebra", better understand the process of the plane rectangular coordinate system from "single axis" to "double axis", grasp the mathematical nature of the plane rectangular coordinate system, and lay the foundation for the formation of a new "schema" in the following new environment. Finally, through real life, students are inspired to apply the "diagram" formed by themselves to the real world, and train students to express the world with mathematical language.

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