Development of an RME-Based Hypothetical Learning Trajectory of Least Common Multiple for Elementary School Students

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Abstract

This study aims to develop an RME-based hypothetical learning trajectory of least common multiple for elementary school students. This study used research & development based on the Plomp development model. The subjects of this study were 9fourth grade elementary school students in Pekanbaru, Riau. The data of this study was collected by distributing questionaires and conducting interviews. The data collected was analysed by using descriptive and quantitative analysis. The results indicated that the hypothetical learning trajectory developed was very valid with an average score of 84.37% and was very practical with an average score of 89.38% from the students and 88.89% from the teachers. Therefore, the RME-based hypothetical learning trajectory of least common multiple can be further used by elementary school students and teachers.

1. INTR0DUCTION

Mathematics is a discipline that has a broad and unlimited scope. In everyday life, mathematics is often used to solve several problems. To introduce mathematical concepts to students, teachers must design good mathematics learning so that students are able to understand concrete things. According to the National Council of Teachers of Mathematics (NCTM) (Rawa et al., 2016) mathematics learning should enable students to: (1) recognize and use mathematical ideas, (2) understand how mathematical ideas are related and build on each other to produce a coherent unit, (3) recognize and apply mathematics in other contexts. Thus, the mathematical connection is the relationship between situations, problems, and mathematical ideas in certain concepts or procedures.

Ideal learning certainly cannot be separated from the planning process of learning. According to the Ministry of National Education, the objectives of elementary mathematics learning are 1) understanding mathematical concepts, explaining the interrelationships between these concepts and then applying the concepts/algorithms in a flexible, accurate, efficient and precise way in

solving problems, 2) using reasoning on patterns and traits, performing mathematical manipulations in making generalizations, constructing proofs or explaining mathematical ideas and questions; 3) solving problems including the ability to understand problems, designing mathematical models, completing models, and interpreting the solutions obtained; 4) communicating ideas with symbols, tables, diagrams or other media to explain the situation or problem; 5) having an attitude of appreciating the use of mathematics in everyday life (Surya, 2018). From these objectives, it can be concluded that mathematics has an important role for students in the development of science and technology.

To achieve these learning objectives, it certainly cannot be separated from the role of the teachers to design appropriate and ideal learning (Alim et al., 2020; Sari et al., 2020). Therefore, teachers are required to be able to design a good learning. The lesson plan is a teacher's detailed planning of the course of instruction or "learning trajectory" for a lesson. However in practice, the lesson plan is a mere formality (Fauzan et al., 2013). Mathematics learning that is oriented to the mathematization of everyday experiences is Realistic Mathematics Education.

2. THEORETICAL FRAMEWORK
a. Realistic Mathematics Education
Realistic Mathematics Education (RME) is a well-known learning approach from Netherlands developed by Freudenthal. Students have opportunities to share their strategies and inventions with each other. Mathematics can be developed from (personal) reality in natural manner: the formal rules and procedures can be derived from the informal working methods of the pupils - that is the heart of didactic realism (Treffers, 1993).

RME is learning that is adapted to the social, cultural, and community characteristics. In RME, the principle is that students must develop their own mathematical models. Understanding of mathematical concepts, principles, and facts is obtained from what the students themselves find in everyday life (Lestari et al., 2019). This is intended to make students understand more deeply and assume that mathematics is not a collection of formulas that must be memorized.

According to the Ministry of National Education, RME is a learning concept that helps teachers relate the instructional materials being taught to students' real-world situations and helps teachers to encourage students to make connections between their knowledge and its application in their lives as family and community members (Yulia et al., 2020) (Alim et al., 2020). This is expected to give meaning to students.

There are six principles of RME (Alim: 2020):

a. Reality principle

As in most approaches to mathematics education, RME aims at enabling students to apply mathematics. The overall goal of

mathematics education is that students
must learn to use their mathematical understanding and tools to solve problems. In RME, however, this reality principle is not only recognizable at the end of the learning process in the area of application; reality is also conceived as a source for learning mathematics.

a. Level principle
The level principle guides growth in mathematical understanding and that it gives
the curriculum a longitudinal coherency. This long-term perspective is characteristic of RME. There is a strong focus on the relation between what has been learned earlier and what will be learned later.

b. Activity principle
The activity principle means that students are confronted with problem situations in which, for instance, they can produce fractions and gradually develop an algorithmic way of multiplication and division, based on an informal way of working.

c. Guidance principle
In RME both the teachers and the educational programs have a pro-active role. They steer the learning process, but not in a fixed way by demonstrating what the students have to learn.

d. Interaction principle
Within RME, the learning of mathematics is considered to be a social activity. Education should offer students opportunities to share their strategies and inventions with each other.

e. Intertwinement principle
In RME that mathematics, as a school subject, is not split into distinctive learning
strands. From a deeper mathematical perspective, the domains within mathematics cannot be separated. They are all linked to each other.

Gravemeijer (Mendrofa, 2017) states that there are three main principles of RME that are (1) guided reinvention and progessive mathematizing, (2) didactical phenomenology, and (3) self-developed model. The three principles are operationalized into five basic characteristics of RME that are (1) phenomenological exploration or the use of contexts, (2) the use of modelsorbridging by vertical instrumens, (3) the use of students own productions and constructions or student contribution, (4) the interactive character of the teaching process or interactivity, and (5) the intertwining of various learning standards.

b. Learning trajectory

A learning trajectory is a series of activities that students perform in solving a problem to understand a concept. The learning trajectory is to describe the scheme of a lesson that will be taken by students consisted of some steps. Clemements and Sarama state that a learning trajectory has been defined as "descriptions of children's thinking and learning in a specific mathematical

domain, and a related conjectured route through a set of instructional tasks designed to engender those mental processes or actions hypothesized to move children through a developmental progression of levels of thinking" (Atsnan , 2018; Wilson et al., 2014).

According to Simon (in Wijaya, 2004) there are three main components of a hypothetical learning trajectory that are learning objectives, learning activities, and hypothetical learning process. Learning objectives are very useful to determine the direction and learning strategies that will be applied. Learning objectives are also useful as foundation to decide what activities should be performed to achieve the desired learning objectives. The hypothetical learning process is useful to determine actions to overcome various problems that students may face later.

Hypothetical learning trajectory was first introduced by Simon (1995). A hypothetical learning trajectory can be used by teachers to determine and formulate learning objectives to be achieved, and then the teacher also makes strategic steps that will be used to realize these objectives (Nurdin, 2011). With a hypothetical learning trajectory, teachers can also develop students' thinking skills and provide learning activities.

The study conducted by Andrews-Larson et al. (2017) develops a hypothetical learning trajectory that draws on three instructional design heuristics of RME that are:

a. An instructional sequence should be based on experientially real starting points.
b. The task sequence should be designed to support students in progressing toward a set of mathematical learning goals associated with the instructional sequence
c. Classroom activity should be structured so as to support students in developing models-of their mathematical activity that can then be used as models-for subsequent mathematical activity

Developing learning models or methods can be done by designing a hypothetical learning trajectory that is able to develop students' informal and formal abilities (Suciana et al., 2020). A hypothetical learning trajectory is a series of activities that will be passed by students to solve problems or understand a concept (Atsnan, 2018). A hypothetical learning trajectory consists of the goal for the students' learning, the mathematical tasks that will be used to promote student learning, and hypotheses about the process of the students' learning. A hypothetical learning trajectory gives information about the learning process that will be undergone and the concepts that will be learned in each step.

In addition, a hypothetical learning trajectory must be developed based on a learning approach that is suitable for students one of which is RME. RME emphasizes important two things are a

hypothetical learning trajectory and development of learning models. RME is student-centered learning approach that the teachers have a role as a facilitator, and contextual problems become the focus of learning (Suciana et al., 2020; Alim et al., 2020). Pasinggi and Thuken (2019) state that the learning activities in RME provide students with a stimulus for learning. A hypothetical learning trajectory is very important to determine learning routes that bring students to achieve learning objectives. If students understand the routes, they will be able to solve the given problems.

In general, the instructional materials on the LCM include the instructional materials of multiples, common multiples, and the LCM. These instructional materials are important in everyday life. This is why in teaching these instructional materials teachers should make sure that students understand it clearly and correctly so that they are able to apply the instructional materials to solve problems in their lives. Teachers teach the instructional materials on the LCM together with the instructional materials of the GCD. Some students find it difficult to understand. Students who do not understand the concept of the LCM will always make mistakes in solving problems that require the LCM. This shows the importance of experience-based activities to understand the concept of the LCM. By using the RME approach, students' informal knowledge about the LCM will be used as a bridge for students to solve the given problems. In principle, RME emphasizes that

mathematics is not an object to be transferred to students but as a form of human activities. The hypothetical learning trajectory developed in this study can be a guideline for determining the learning activities to achieve the desired goals. In this study, the researcher develops an RME-based hypothetical learning trajectory of instructional materials on the LCM.

3. METHOD

This study is Research & Development that aims to develop an RME-based hypothetical learning trajectory of instructional materials on the LCM to improve students' understanding of the concept of the LCM. This study used the Plomp development model. The Plomp model is considered more flexible compared to other development models because the activities in each phase can be adapted to the characteristics of the study (Arianatasari & Hakim, 2018).
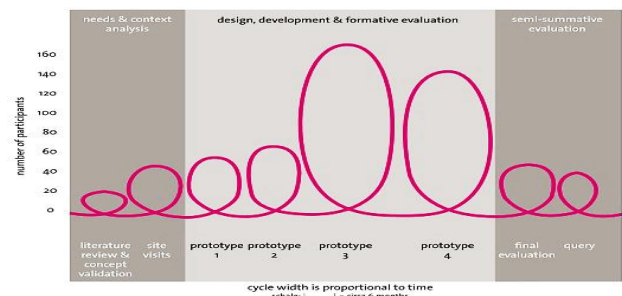

Figure 1. The phase in the Plomp development model

Plomp states that there are three phases in the Plomp development model that are the preliminary research phase, the development phase, and the

assessment phase. This research was conducted on fourth grade elementary school students.

1) In the preliminary research phase, the researchers conducted some analyses that are needs analysis, curriculum analysis, student analysis, and instructional material analysis.
2) In the development phase, the researchers developed the initial product and conducted expert evaluation, one-to-one evaluation, and small group evaluation.
3) In the assessment phase, the researchers conducted two tests that were validity and practicality test.

Data in this study was obtained from experts, and the data contained information about how acceptable the developed hyphotetical learning trajectory. Data in this study was also obtained from the teachers and students, and the data contained information about how practical the developed hyphotetical learning trajectory. The practicality test was conducted in 2 stages, one-to-one and small group evaluation. In the one-to-one evaluation, there were three students with mixed-abilities. In the small groupe valuation, there were six students with mixed-abilities.

4. RESULTS AND DISCUSSION

The results of this study that consist of the results in the preliminary research phase, the development phase, and the assessment phase are presented below.

a. Results in the preliminary research phase

In the preliminary research phase, the research a rough draft of the hypothetical learning trajectory based on some analyses. The draft consisted of five learning objectives, and the five learning objectives are presented below.
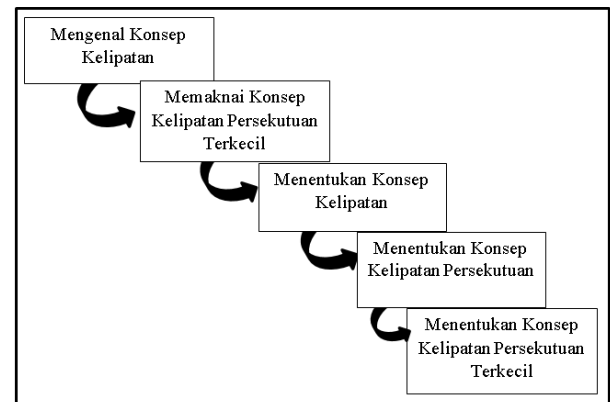

Figure 2. the rough draft of the hypothetical learning trajectory

To achieve the learning objectives, the researcher provided four learning activities. The learning activities are presented below.

Activity 1: Get to know the concept of multiples through the making of sago noodles. The purpose of this activity was that students were able to understand the concept of multiples. The concepts of multiples had to be understood by students in order to be able to learn and determine the LCM of several numbers. This activity was carried out by providing opportunities for students to be a birthday girl that was cooking sago noodles for her friends who come on her birthday. One pack of sago noodles were for 3 portions of fried sago noodles or 5

portions of sago noodles soup. The students were asked to determine how many packs of sago noodles should be bought by the girl to make the same number of portions of fried sago noodles and sago noodles soup. First, the students were asked to determine the multiples of 3 and 5. Second, they were asked to determine the LCM of the two numbers. Third, the LCM was divided by 3 and 5. Finally, the results of the two processes of division were added.

Activity 2: Interpret the concept of the LCM and determine the concept of multiples through the making of asidah cake. The purpose of this activity was that students were able to interpret the concept of the LCM and determine the concept of multiples. This activity was carried out by providing opportunities for students to be a girl that was cooking asidah cake for a feast. First, the students were asked to draw four portions of asidah cake which each portion had eight petals. Second, the students were asked to total all petals drawn before. Third, the students were asked to total all petals if there were 15 portions of asidah cake. The students had to understand the concept of multiples of numbers to determine common multiples of two numbers.

Activity 3: Determine the concept of common multiples through the arragement of a dance practice schedule. The purpose of this activity was that students were able to determine the concept of common multiples. This activity was carried out by providing opportunities for students to be two girls that were members of a Malay dance studio, and they had two different dance practice schedules that were every 2 days and every 3 days. The students were asked to determine when they would meet again if they met at the dance studio today. First, the students were asked to determine the dance practice days of the first girl by ticking the correct box. Second, the students were asked to determine the dance practice days of the second girl by ticking the correct boxes. Third, the students were asked to determine the dance practice days of both girls by ticking the correct boxes

Activity 4: Determine the concept of the LCM through a word problem of two oil palm truck. The purpose of this activity was that students were able to determine the concept of the LCM. This activity was carried out by providing opportunities for students to be Pak Bono who had two oil palm trucks. The first oil palm truck stopped every 50 kilometers, and the second oil palm truck stopped every 60 kilometers. The students were asked to determine at what kilometers both trucks stopped together. First, the students were asked to determine at what kilometers the first truck stopped. Second, the students were asked to determine at what kilometers the second truck stopped. Third, the students were asked to determine at what kilometers both trucks stopped together.

b. The results in the development phase

In this phase, the researcher developed the initial product of the hypothetical learning trajectory and conducted expert evaluation, one-to-one evaluation, and small group evaluation. The expert stated that the hypothetical learning trajectory was valid. The results of the validity test by the experts are presented below.

Table 1. The validity test results

No.	Aspects	Percentage	Interpretation
1	Instructional Materials	83,12 %	Very Valid
2	Language	80 %	Very Valid
3	Design	90 %	Very Valid
	Average	**84,37%**	**Very Valid**

After that, the researcher conducted one-to-one evaluation to three students with mixed-abilities. This one-to-one evaluation aimed to find out the practicality of the hypothetical learning trajectory. The researcher then conducted small group evaluation to six students with mixed-abilities. This small group evaluation was conducted in two meetings. The results of both evaluations are presented below.

One-to-one evaluation

In the one-to-one evaluation, the researcher found that overall students were able to understand the concept of the LCM and determine the LCM of some numbers. Moreover, the hypothetical learning trajectory applied succeeded in providing the right concept of the LCM for students. However, there was a student that solved the given contextual problems slowly so that the student was assisted by giving the student simple questions to help the student in deciding how to solve the problems. Based on the

problem, the researcher added some actions the teachers should do to cope with students' learning problems into the hypothetical learning trajectory.

Small group evaluation

The small group evaluation was carried out in two meetings. The evaluation was participated by six students with mixed-abilities. The results of the small group evaluation are presented below.

In activity 1, at first students read a word problem, and they did not have an idea to solve the problem. Therefore, probing questions were given to encourage deep thought about the problem. In this activity, students were expected to know the concept of multiples through the making of sago noodles. The results showed that there were several students who were able to quickly determine how many packs of had to be bought. However, there were also several students who did not understand the concept, but they gave

the right answers. Overall, students knew the concept of multiples and were able to solve the word problem with the right concept of multiples.

In activity 2, the students were asked to total all petals if there were 15 portions of asidah cake. The results showed that the students answered the word problem correctly eventhough they did it in different ways. The results indicated that the given contextual problems was able to encourage the students to understand the initial concept to determine multiples. Because the purpose of this activity was that students were able to interpret the concept of the LCM and determine the concept of multiples. The students were expected to be able to determine the multiples of the number of petals of asidah cake in each portion.

In activity 3, the students were asked to determine when two girls that were members of a Malay dance studio and had two different dance practice schedules that were every 2 days and every 3 days would meet again if they met at the dance studio today. The results showed that the students understood the given problem, and they were able to determine the common multiples of two numbers. In conclusion, the contextual problem was able to lead students to understand the concept of common multiples of two numbers.

In activity 4, the students were asked to determine at what kilometers both trucks would stop together if the first truck stopped every 50 kilometers and the second truck stopped every 60

kilometers. The results showed that the students were able to answer and understand the contextual problem because students understandood the concept of the LCM. In conclusion, the contextual problem was able to lead students to the concept of the LCM of two numbers.

The results in the assessment phase

After the small group evaluation was conducted, the researcher distributed some questionnaires to the students and teachers to assess the practicality of the hyphotetical learning trajectory. The results showed that the hyphotetical learning trajectory got an average practicality score of 89.38% from the students and 88.89% from the teachers that indicated a very practical hyphotetical learning trajectory.

Table 2. The Practicality Result

No	Result	Average
1	Practicality from Teachers	88,89%
2	Practicality from Students	89,88%
	Average	**89,38%**

There were 8 aspects assessed by the teachers in the practicality test . The first aspect was whether the instructions in each activity were clear and easy to understand. The second aspect was whether the objectives were in accordance with the indicators. The third aspect was whether the syntax provided contextual problems that supported

mathematics learning and were easily implemented by teachers. The fourth aspect was whether the instructional materials were characterized by RME, and the learning activities were in accordance with the syllabus. The fifth aspect was whether the worksheet encouraged students to apply RME learning and also the questions were easy to understand by students. The sixth aspect was whether the hyphotetical learning trajectory was written well so that students easily understood the content. The seventh aspect was whether the hyphotetical learning trajectory had attractive appearance. The eighth aspect whether the hyphotetical learning trajectory provided many benefits for students.

There were 7 aspects assessed by the students in the practicality test. The aspects were the same as the aspects assessed by the teachers but without the aspect of syntax.

c. Discussion

HTL can be used if you have paid attention to the components and characteristics of the RME learning carried out. As for the results of the assessment of the HLT that has been developed for aspects of content, objectives, language and educational technology by the validator, data is obtained with a percentage of 84.37% with a valid category and has been assessed for practicality by students and also teachers with a very practical category with the percentage of teachers

being 89.38 % and the percentage of students 88.89%.

Based on the results of the study and the results of data analysis after treatment, it was concluded that HLT can be used in Mathematics learning on LCM material. The HLT developed has been designed according to contextual problems that exist around students (for example see Putra et al., 2011) and also uses language that is easily understood by students, so that students can use HLT and develop contextual questions that are done and find out the solutions.

5. CONCLUCION

This study is Research & Development which is to produce an RME-based hypothetical learning trajectory of instructional materials on the LCM. From the results and discussion, it can be concluded that the hypothetical learning trajectory is very valid and practical. The validity test result showed that the hypothetical learning trajectory was very valid with the average percentage of 84.37%. Moreover, the practicality test result showed that the hypothetical learning trajectory was very practical with the average percentage of 89.38% from the students and 88.89% from the teachers. In conclusion, the RME-based hypothetical learning trajectory of instructional materials on the LCM can be used by elementary students and teachers.The HLT can be used for teachers in the teaching and learning process, which is alsi expected to be able to develop better by paying attention

that are around to achieve the desired goals. For further researchers, it is recommended to try out in large groups in order to get better student learning outcomes.

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