

The Ethnomathematics of Chinese Tulou Building Architecture as Geometry Teaching Material in Elementary School

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Abstract

The purpose of this study is to study ethnomathematics aspects contained in Chinese Tulou architecture. The study uses the qualitative research method to describe the ethnomathematics value of Tulou building in China, and then connect Chinese Tulou building with mathematics. The method of data collection is document's content analysis. The study found that Chinese Tulou contains a wealth of mathematical knowledge, including mathematical knowledge of geometric figures in mathematics, such as circle, triangle, rectangle, square, octagon, cylinder, octaprism, as well as the area and volume of geometric figures. Therefore, in daily teaching life, teachers can use Tulou as teaching materials, which was not only help students understand mathematics knowledge but also improve the enthusiasm of students.

Keywords: *Chinese Tulou buildings; elementary school; ethnomathematics*

1. INTRODUCTION

Many researchers said that mathematics is influenced by various environments (Evans, 2004; Kubiato et al., 2010; Lei & Zhao, 2007), including language (Cai et al., 2014; Duval, 2006), religion (Suryanti et al., 2019), technology (Aixia et al., 2020; Chotimah et al., 2020; Wijaya, Jianlan, et al., 2020; Zhang et al., 2020) and society (Moffat & Shabalina, 2017; Pramudita & Rosnawati, 2019; Saul, 2011). National Council of Teachers of Mathematics (NCTM) refers to the relationship between mathematics education and social-cultural reality.

Therefore, mathematics is closely related to national culture. China has many building which are closely related to mathematics to study. China has a wall of china that was built by more than 6 different kingdoms and is more than 2300 years old. In previous research, Dwidayati et al explored China's great wall in the perspective of ethnomathematics and found many things that can be learned from old buildings in China (Dwidayati et al., 2019). In this study, researchers continued previous research to explore

other buildings in China with an ethnomathematics perspective.

This time elementary school students still think that mathematics has no relationship with everyday life (Alim et al., 2020; Tamur et al., 2020). Teachers must have Pedagogical Knowledge to bring objects in real life into the world of mathematics (Fitriani et al., 2018; Putra, 2019; Putri & Zulkardi, 2018; van den Heuvel-Panhuizen & Drijvers, 2014). Lin and colleagues make mathematics an interesting subject and change students' perspective who say that mathematics is a difficult and boring subject (Lin et al., 2020; Wijaya, Hidayat, et al., 2020; Wijaya, Ying, et al., 2020). Ethnomathematics is one way to improve students' mathematical abilities (Dwidayati et al., 2019; Rosa & Gavarrete, 2017).

There are advantages when ethnic mathematics is used mathematics curriculum:

- can make students understand the role of mathematics in all kinds of society, and then understand that mathematics comes from people's actual needs (Lubis et al., 2021).
- Learn to appreciate the achievements of different cultures and be proud of their cultural heritage (Achor et al., 2009).
- make learning more meaningful by linking mathematics with history, language, art and other topics (Umbara & Prabawanto, 2021).
- Integrating the cultural heritage of ethnic minorities into the curriculum can build the self-

confidence of ethnic minority students

- make students more interested in mathematics (Pramudita & Rosnawati, 2019).

Ethnomathematics is a kind of mathematical knowledge related to students' cultural background, helping students better understand mathematical knowledge (Andy Rudhito et al., 2020; Putra et al., 2020; Witri et al., 2019). From the results of previous research, it can be seen that ethnomathematics has a good effect and can continue to be developed as a good and interesting learning approach.

Chinese Tulou architecture is a kind of special architecture for resisting wild animals and robbers in the mountains and forests and embodying Confucianism, which is the ideal for a big family to live together (Li et al., 2012). It is round, semicircular, square, quadrangular and pentagonal (figure 1). The formation of Tulou dates back to the time when the Hakka people system was formed in Fujian, Guangdong and Jiangxi's border area in the late Tang Dynasty and early Song Dynasty in 1960s and 1970s matured in the late Ming Dynasty, Qing Dynasty and the Republic of China. Earth building is the product of Hakka ancestors in inheriting and carrying forward Chinese traditional culture, and the crystallization of Hakka ancestors' wisdom from generation to generation (Luo et al., 2019). The large-scale hakka earth building is the "Big Mac" in the type of mountain residential buildings. hakka earth building can be

called the "aircraft carrier" in ancient residential buildings, and known as the "oriental ancient castle". In July 2008, Tulou building (figure 2) and Yongding

hakka in Fujian China and succeeded in becoming a UNESCO world heritage (Huang, 2020; Sun et al., 2014).

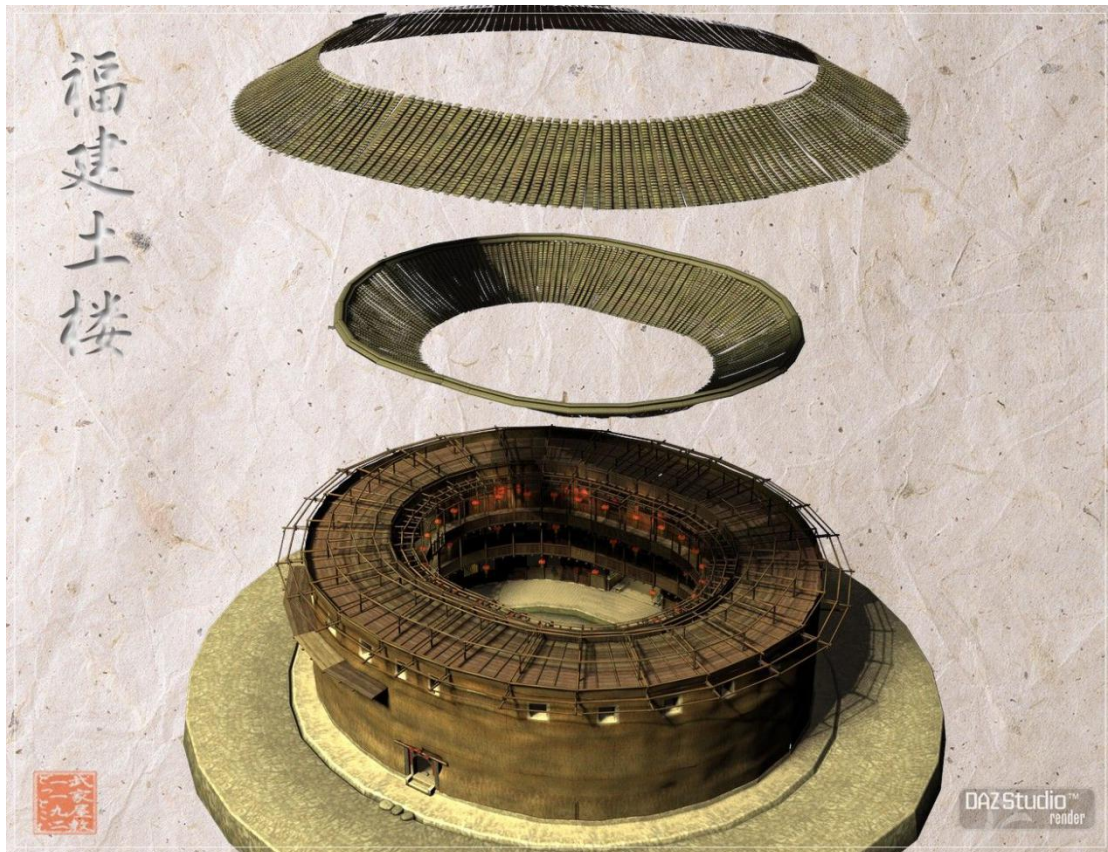


Figure 1. Structure of Tulou Building (From Google)



Figure 2. Tulou Building, Fujian, China (From: Google)

The shape of Tulou reflects the knowledge of geometric figures in mathematics. The area and volume of these geometric figures show that Chinese Tulou architecture contains interesting mathematical knowledge, and Tulou itself is a cultural product with national characteristics, reflecting the ancient Chinese people's emotion and wisdom.

The main aim of this study is to explore the mathematical knowledge of Tulou culture and to provide reference for ethnomathematics values.

2. METHOD

This study was conducted from December 20, 2020 to March 23, 2021. This study mainly studied the National Mathematical Culture of two Tulou buildings in China, namely, Chengqi building in Fujian Province, Daoyun building in Guangdong Province. This study used the qualitative research method to describe the ethnomathematics value of Tulou

building in China, and then connect Chinese Tulou building with mathematics. The method of data collection is documents' content analysis. The data is obtained by consulting the library's paper literature and the official authoritative information on the Internet, which focus on research about the geometric value of the Tulou building.

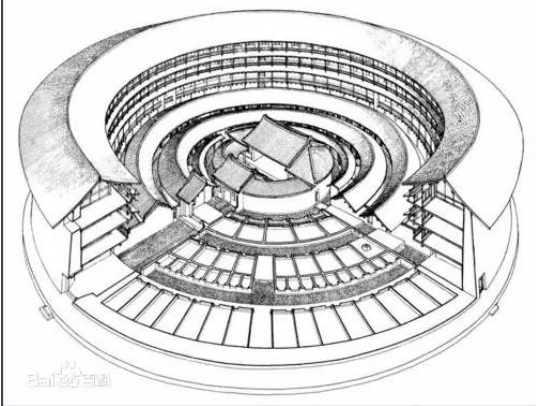
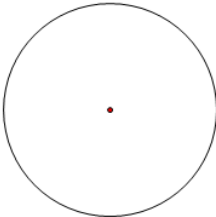
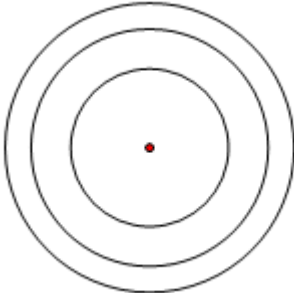

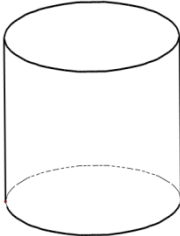


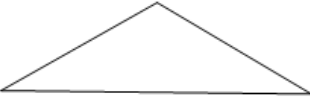
3. RESULTS AND DISCUSSION

Through the literature survey of Tulou, we knew that Tulou contains interesting mathematical geometry knowledge. In the following, a detailed explanation about this is provided.

a. Ethnomathematics of Chengqi building

Chengqi building, also known as "king of earth buildings", was built in the first year of Chongzhen in Ming Dynasty. It took 81 years to complete. This earth building has the largest number of rings and scale among Hakka round earth buildings.

Table 1. Explore geometry aspects of Hakka round earth buildings

The Figure of Tulou Buildings	Geometry Values
	
<p>Chengqi building</p>	
	
<p>vertical view of Chengqi building</p>	
	
<p>Close up view of Qiyun building</p>	

From table 1 it can be found that the Chengqi building is related to the circle of a mathematical geometric figure and the transformation of a mathematical figure.

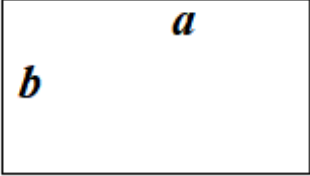

We can see that the Chengqi building is related to the cylinder in the solid mathematical geometry. We can find that the building is related to the triangle and

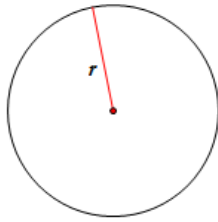
rectangle in the mathematical plane geometry.

Chengqi building is also related to the mathematical knowledge of geometric figure area and volume. From the data

the researchers obtained from the Chengqi building, the researchers made math problems. which is introduced in detail is as follows:

Table 2. Example of plane geometry problems based on tulou building

Questions	Answer
<p>What the reason the Chengqi building is a circle? Is that the area of plane figure in mathematics is related to perimeter knowledge?</p> <div style="text-align: center;">  </div> <p>a = Length b = Width l_1 = Perimeter S_1 = Area of Rectangle</p> <div style="text-align: center;">  </div>	<p>If a circle, a square, and a rectangle have the same perimeter, the circle has the largest area, and the proof is as follows:</p> <p>In the case of rectangles: Let a be the length of the rectangle and b the width of the rectangle, l_1 be the perimeter of the rectangle and S_1 be the area of the rectangle.</p> $\because l_1 = 1, \therefore a + b = \frac{1}{2}$ <p>if $a = \frac{1}{3}$, that $b = \frac{1}{6}$,</p> $\therefore S_1 = \frac{1}{18}$ <p>In the case of square: Let a be the side length of a square, l_2 be the circumference of a square and S_2 be the area of a square.</p> $\because l_2 = 1, \therefore a = b = \frac{1}{4}$ $\therefore S_2 = \frac{1}{16}$



In the case of circle:
 Let the radius of the circle be r , the circumference of the circle be l_3 , and the area of the circle be S_3 .

$$\because l_3 = 1, \therefore r = \frac{1}{2\pi}$$

$$\therefore S_3 = \frac{1}{4\pi}$$

So the circle, square and rectangle with the same circumference, and the circle area is the largest, which means that the same amount of building materials can get more spacious courtyard space. Therefore, the shape of Tulou is mainly round rather than rectangular. That's why the building was original, not rectangular.

Table 3. Example of polyhedron based on tulou building

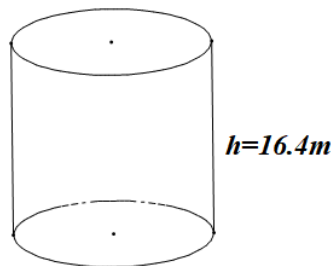
The whole Chengqi building can be regarded as a cylinder. According to the data, the floor area and height of the Chengqi building are 5376.17 and 16.4 meters. what is the volume of Chengqi

Let the bottom area of the cylinder be S , the height be h , and the volume be V .

$$\therefore S = 5376.7m^2, \quad h = 16.4m$$

$$\therefore V = Sh = 5376.7 \times 16.4 = 88177.88m^3$$

So the volume of Chengqi building is about $88177.88m^3$.



building?

b. Ethno mathematics of Daoyun building

Daoyun building was built in 1477 and took more than 100 years to complete. It is the largest octagonal earth building in China. It can be seen from table 3 that

Daoyun building is related to octagon, circle, transformation of figure and prism.

Daoyun building looks like a regular octagonal prism, but the overall Daoyun building shape is arranged according to

the diagram of the Chinese Eight trigrams universe (see figure 3). The circular open space in the middle of the building symbolizes the Taiji of the Chinese Eight trigrams universe diagram. The left and right public wells of the circular open space in the middle of the

building symbolize the two instruments of the diagram of the Chinese Eight trigrams universe, commonly known as yin and yang, and the eight-sided rooms of the building are like Chinese the Eight Diagrams.



Figure 3. The diagram of Chinese Eight Trigrams universe

The Eight trigrams were created by Fuxi in ancient China and are the crystallization of ancient Chinese wisdom. In April 1703, Leibniz, the founder of the binary system in mathematics, received a long letter from the missionary Bai Jin in November 1701. He saw the attached sequence diagram and square diagram of

Fuxi 64 hexagrams in China. A few days later, Leibniz published the explanation of binary arithmetic and used binary to explain the hexagram of Fuxi. What is the relationship between Chinese eight trigrams and binary in mathematics? The following is a detailed introduction.

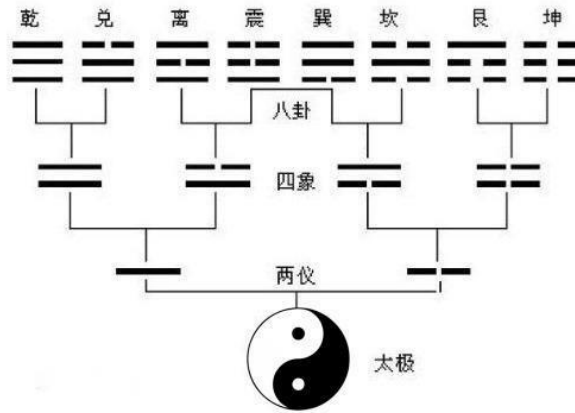


Figure 4. Binary tree in mathematics

The Eight Diagrams is composed of two basic symbols. The process of formation can be represented by binary tree in mathematics (see Figure 4). These four symbols are collectively called "four images". These eight symbols are called "Qian", "Dui", "Li", "Zhen", "Xun", "kan", "gen" and "Dui" respectively in China, and they are collectively called "Bagua". If each line represents 1, then the symbol can be expressed in binary

form. If the binary codes is converted to a decimal, the table 4 can be obtained. It can be seen from the table that the order of positions from right to left of the hexagram is just corresponding. For example, in Zhouyi, the number of positions from right to left of the hexagram is "7", while the number of positions from right to left of the hexagram is just "7".

Table 4. the Eight Trigrams and binary codes

Symbol	☰	☱	☲	☵	☴	☳	☶	☷
Mean	111	110	101	100	011	010	001	000
Decimal	7	6	5	4	3	2	1	0

In the book of changes, 64 hexagrams can be obtained by combining the eight trigrams, and the arrangement of the 64 hexagrams is shown in the order in the circle (see Figure 5). Binary codes

similarly represent hexagrams, then the order of 64 hexagrams just corresponds to the number that converts binary codes into decimal.

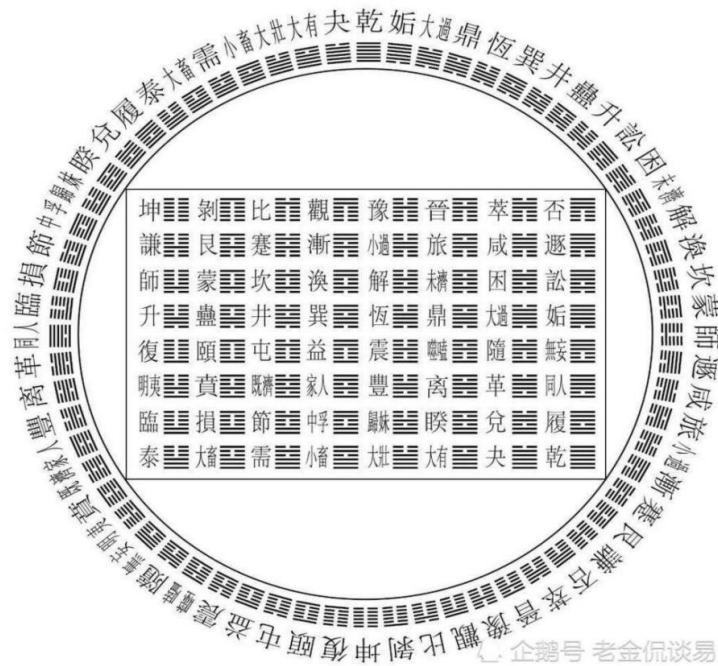

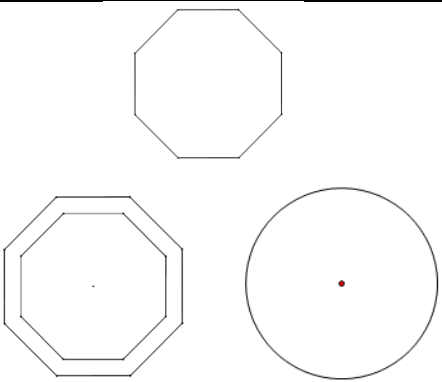
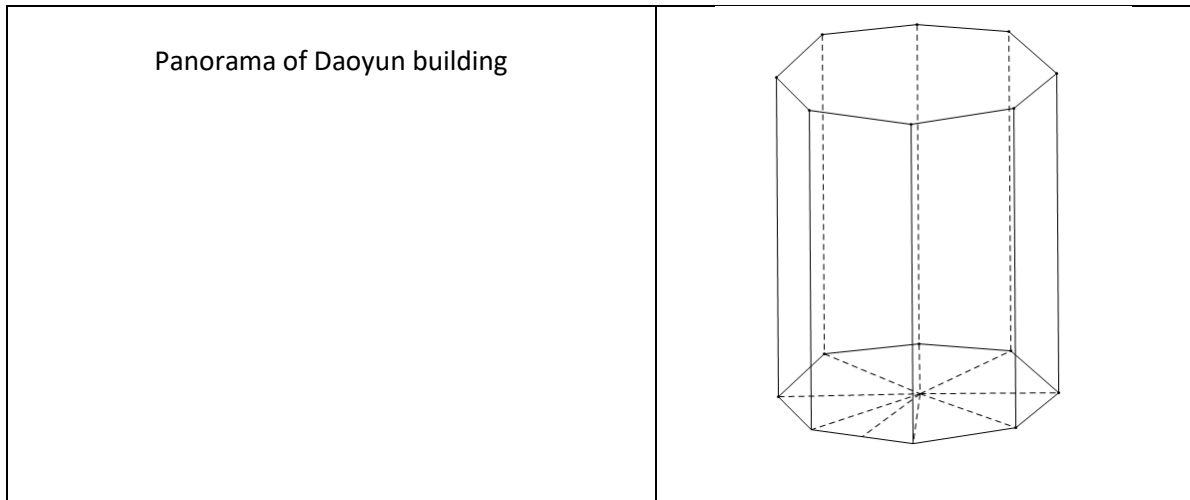


Figure 5. 64 diagrams of China

From the above analysis, we can see that Chinese hexagram is really closely related to binary codes.

Table 5. Exploration Results on Ethnomathematics aspects of Daoyun building

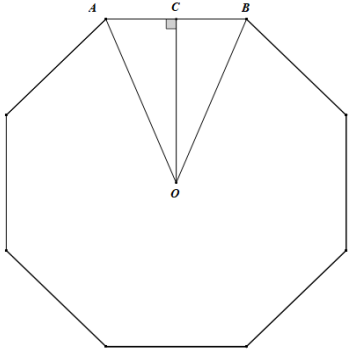
The Figure of Daoyun Building	Geometry Values
 <p data-bbox="370 1812 743 1848">Aerial view of Daoyun building</p>	



Daoyun building is also related to the mathematical knowledge of area and volume in geometry, which is explained in detail below. The area of regular

octagon can be obtained according to the knowledge of plane geometry. The specific solving process is as follows:

Table 6. An example of finding the octagon area of a Tulou building

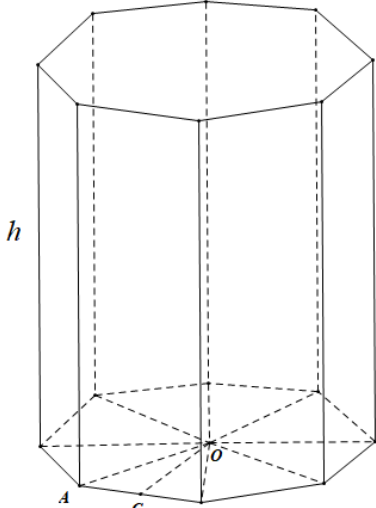
<div style="text-align: center;">  </div> <p>The perimeter of Daoyun building is 328 meters. That is, the perimeter of the regular octagon is 328 meters. What is the area of the octagon from the Daoyun building image above?</p>	<p>Let the perimeter of a regular octagon be L, and the area of a regular octagon be S. Draw a perpendicular OC through the center O of the regular octagon and perpendicular to the side length ab of the regular octagon.</p> $\therefore L = 328$ $\therefore AB = 328 \div 8 = 41$ $AC = 41 \div 2 = 20.5$ $AO = \frac{AC}{\sin \angle AOC}$ $\therefore \sin \angle AOC = \sqrt{\frac{1 - \cos \angle AOB}{2}} = \frac{\sqrt{2 - \sqrt{2}}}{2}$ $AO = 20.5 \times \frac{2}{\sqrt{2 - \sqrt{2}}}$ $\therefore S = 8 \times \frac{1}{2} AO \cdot BO \sin \angle AOB$ $= 8 \times \frac{1}{2} \times 20.5 \times \frac{2}{\sqrt{2 - \sqrt{2}}} \times 20.5 \times \frac{2}{\sqrt{2 - \sqrt{2}}} \times \frac{\sqrt{2}}{2}$ $= 8112.403m^2$
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	Therefore, the area of octagon is $8112.403m^2$
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For an example of finding the volume of an octagon-shaped tulou building can

be seen in table 7, The specific solution process is as follows:

Table 7. The volume of an prism-shaped Tulou building

<p>through consulting the data, it can be seen that the height of the Daoyun building is 11.5m, so the height of the positive octagons is 11.5m, and the perimeter of the bottom is 328m.</p> 	$\begin{aligned} \therefore S_{\square AOB} &= \frac{1}{2} AO \square BO \sin \angle AOB \\ &= \frac{1}{2} \times 20.5 \times \frac{2}{\sqrt{2-\sqrt{2}}} \times 20.5 \times \frac{2}{\sqrt{2-\sqrt{2}}} \times \frac{\sqrt{2}}{2} \\ \therefore V &= 8 \times S_{\square AOB} h \\ &= 8 \times \frac{1}{2} \times 20.5 \times \frac{2}{\sqrt{2-\sqrt{2}}} \times 20.5 \times \frac{2}{\sqrt{2-\sqrt{2}}} \times \frac{\sqrt{2}}{2} \times 11.5 \\ &\approx 93292.632 \end{aligned}$ <p>So the volume of regular eight angular prism is $93292.632m^3$.</p>
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Based on the exploration of Tulou building, we can see that ethnomathematics are related to elementary school students' daily lives. Elementary school teachers can introduce geometric shapes and make practice questions using the Tulou Building. The form of questions from the exploration of Tulou building is very interesting.

4. CONCLUSION

Based on the research, we found that Tulou architecture has interesting mathematical and cultural knowledge. For example, most Chinese Tulou architecture is circle rather than rectangular, because the area of the circle is the largest among the circles and rectangles with the same perimeter. In this way, using the same materials, we can get a larger area. In addition, the reason why Daoyun building presents the

shape of octagonal is that Daoyun building was originally designed according to the diagram of Chinese Eight trigrams universe. The diagram of Chinese Eight trigrams universe itself contains binary mathematical knowledge, and the inventors of binary codes also use binary codes to explain Chinese eight trigrams.

Therefore, in daily teaching, teachers can use these interesting and strong national cultures as students' learning materials, helping students better grasp knowledge and improve students' strong interest in learning mathematics.

This research is limited by using information on the internet. Researchers have not go to Tulou Building, Fujian, China to explore ethnomathematics in Tulou building more deeply. As suggestions for further research, there are still many historical buildings containing ethnomathematics in various countries. Thus, teachers can use ethnomathematics to improve students' mathematical abilities.

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