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Elementary Teachers' Knowledge on Fraction Multiplication: An Anthropological Theory of the Didactic Approach

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Abstract

This study aims to evaluate Indonesian in-service elementary teachers' knowledge on fraction by their responses to contextual problems on multiplication of a fraction by a whole number (O_1) and multiplication of a fraction by a fraction (O_2). Fifty Indonesian in-service elementary teachers from Elementary School for Teacher Education study program, University of Riau, participated in this research in 2015. This research is based on anthropological theory of the didactic (ATD), in particular *praxeology*, as a framework to analyse the data. The result showed that only 22% of Indonesian in-service elementary teachers have a satisfactory relation to O_1 and 64% of them have succeeded on the problems related to O_2.

Keywords: *Anthropological theory of the didactic, fraction multiplication, in-service elementary teachers, contextual problems*

1. INTRODUCTION

Research about pre-service and in-service elementary teachers' knowledge on fractions has been done for some decades. Some studies focus on their capacity to construct word problems for fraction multiplication (Luo, 2009) and fraction division (Leung and Carbone, 2013, Ma, 1999). Luo (2009) found that a significant percentage of the pre-service teachers in U.S were unable to construct appropriate word problems for the given symbolic expressions of fraction multiplication, in term of semantic and unit of measurement. This finding is collateral with the research doing by Ma (1999) that she found most U.S pre-service elementary teachers had difficulties with constructing meaningful problems leading to fraction division. Similarly, Indonesian teachers also have difficulties on operations of fractions (Putra, 2016; Putra & Winsløw, 2018). Putra (2016) found that only 44.53% of Indonesian pre-service teachers could represent multiplication using rectangle models, and only 24.22% of them could do the division of fractions.

The gaps in teachers' knowledge on fractions are caused by generalizing the meaning of multiplication and division from the context of integers to that of fractions operations, which also affects the relationship between those

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concepts (Tirosh, 2000; Isiksal & Cakiroglu, 2011). For instance, the meaning of multiplication as a repeated addition should not be generalized to multiplication of a fraction by a whole number or a fraction by a fraction. These misconceptions probably have their origin at the time when the teachers were themselves students at primary school, and persist through teacher education.

In this study, we are also interested in investigating in-service elementary teachers' knowledge in the sense that it is based on contextual problems on fraction multiplication. We use a different approach to analyse the results. We consider that in-service elementary teachers fail to adopt the relation of subjects X in teacher education institution I to a knowledge object O, which can be analysed more precisely based on an epistemological model, developed with the anthropological theory of the didactic (ATD), called *a praxeology* (Chevallard, 1992). Durrand-Guerrier, Winsløw, and Yoshida (2010) had showed how this model applied to study teachers' individual and collective mathematical and didactical knowledge. More specifically, our study explores the following questions: What are the evaluations of Indonesian in-service elementary teachers' knowledge in designing contextual problems based on ATD? And how does a relation of subjects X in teacher education institution I to knowledge objects O?

2. THEORETICAL FRAMEWORK

Anthropological theory of the didactic (ATD) is a general epistemological model of mathematical knowledge to observe human mathematical activities (Chevallard, 1992). The ATD aims to understand of the ecology of mathematical knowledge that emerges from human practices. The process of transportation of knowledge can be characterised into some object of knowledge (O) and some subjects X supposed to study O that it can be written as R(X, O). The meeting between X and O take place in some institution (I), so the model becomes R_I(X, O). When I is missing, the system reduces to an autodidactic system. This framework will be used to evaluate the relation of X to O in I.

Chevallard (1992) argued that human practice and thinking can be described in term of *praxeology*. A praxeology consists of two interrelated components, a practical block and a knowledge block. The practical block is formed by a type of task (T) and a technique (τ). A type of task (T) is a specific class of problems such as constructing a contextual problem of a given fraction multiplication. The students need a technique (τ) to solve this problem. Then, the knowledge block consists of a technology (θ) used to explain the practical block and a theory (Θ) to justify and reason about the technology (θ). The technology (θ) for the case may, for instance, explain the technique (τ) as based on a part-of a whole relationship, while the arithmetic of fractions is a theory (Θ) to justify the technology. Those four elements (T, τ, θ, Θ) are interdependent.

In relation to study in-service teachers' knowledge on constructing contextual problems on operations of fractions, the main focuses of the

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analysis is the technological discourse underpinning the teachers' written answers (cf. Putra & Witri, 2017). In addition, the teachers become autonomous in the autodidactic system.

3. METHODS

In this study, 50 Indonesian in-service elementary teachers who were taking a bachelor degree at Elementary School for Teacher Education study program, University of Riau, in 2015 participated in designing contextual problems on fraction multiplication. They were in the second year of that program. They had already taken some courses such as fundamentals of mathematics and statistics for education. They also had taken two years diploma course before, but in different areas such as religion and social science education study programs. They had some years of experiences in teaching elementary school students in Riau province.

Those teachers were asked to pose a contextual problem for multiplication of a fraction by a whole number $\left(\frac{1}{2} \times 2\right)$ and multiplication of a fraction by a fraction $\left(\frac{1}{2} \times \frac{3}{4}\right)$. Praxeological reference models for both tasks can be written as follows.

T_1 : given $\frac{a}{b} \times c$, design a contextual problem related to this equation.

T_2 : given $\frac{a}{b} \times \frac{c}{d}$, design a contextual problem related to this equation.

To solve both problems, in-service elementary teachers need some techniques (τ). Since those tasks are more about didactical organization (DO), the techniques can be difficult to describe or show to others. The anthropological approach assumes that any 'way of working', the accomplishment of any task or the resolution of any problem requires the existence of a technique (Barbé, Bosch, Espinoza, and Gascón, 2005). Meanwhile, we can figure out the technologies (θ) such as multiplicative comparison, part-of a whole (for T_1), part of a fraction (for T_2), and multiplication based on measurement of area, and the theory (Θ) to justify θ is arithmetic of fractions.

The data are analysed as relations held by 50 in-service elementary teachers X in I (Elementary School for Teacher Education study program, University of Riau) to certain study objects or didactic stakes, the meaning of multiplication of a fraction by a whole number (O_1) and of multiplication of a fraction by a fraction (O_2). More specifically, O_1 and O_2 have actually been taught to primary school students by the teachers at I (Elementary Schools in Riau), guided by the official curriculum and mathematics text books. The relation of the teachers (X) to object (O) can be written as R(X, O_1) and R(X, O_2). Since X study at the institution I, the specific relation they develop and display there can be written as R_I(X, O_1) and R_I(X, O_2).

4. RESULT

Multiplication a fraction by a whole number

For the task (T_1), there were only 11(22%) in-service elementary teachers designing correct and meaningful contextual problems. Eight of them designed contextual problems based on part of a whole relationship (A1) and the others based on measurement of

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area (A2). Meanwhile, 39 (78%) in-service elementary teachers designed incorrect contextual problems about multiplication of a fraction by a whole number. They designed contextual problems based on repeated addition (A3) (32%), addition of fraction (18%), division of integer (A4) (10%), multiplicative comparison (2%), and no answers (16%). Some examples of their answers are written as follows.

A1*: A father has 2 hectares of land. ½ of this land is given to his cousin. How much land does the father now have?*

A2: *Andi would like to draw his land into a rectangle with ½ m long and 2 m wide. What is the area of the rectangle?*

A3: *Dina has 2 packs of rice. Each pack contains ½ kg of rice. How much rice does Dina have?*

A4: *A sister has 2 apples. Those apples will be given to two of her young brothers. How many apples will be got by each brother?*

Multiplication a fraction by a fraction

In-service elementary teachers performed better in designing contextual problems about multiplication of a fraction by a fraction than multiplication of a fraction by a whole number. Thirty-two (64%) of them were able to pose correct and meaningful contextual problems. Twenty-seven of them design contextual problems based on measurement of area (B1), and the others based on part of a fraction (B2). Meanwhile, 18 (36%) in-service elementary teachers designed incorrect contextual problems about multiplication of a fraction by a fraction. Two of them posed contextual problems based on subtraction of a fraction (B3) and the others either did not give an answer or unclear and unfinished contextual problems. Some examples of their written answers are present as follows.

B1: *A rectangle is ½ m long and ¾ m wide. What is the area of the rectangle?*

B2: *An aunt has ¾ part of cake. ½ of that cake will be given to Ani. How much cake will Ani get?*

B3 4: *A mother wants to make a cake with ½ kg of flour and ¾ kg of sugar. How much other materials are needed if the total weight of the cake should be 3 kg?*

5. DISCUSSION AND CONCLUSION

In-service elementary teachers gave various technologies (θ) for multiplication of a fraction with a whole number than multiplication of a fraction by a fraction, but they displayed several mistakes when designing a contextual problem of a fraction by multiplication. Half of them applied repeated addition and addition as technologies (θ) to explain their contextual problems, but these technology may not appropriate to represent the situation for the multiplication of a fraction by a fraction. It is probably caused by generalizing the meaning of multiplication from the context of integers to that of fractions operations (Tirosh, 2000; Isiksal & Cakiroglu, 2011). It is an evident that

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they have an insufficient relation to the meaning of fraction multiplication. The lack of support from the Institutiton (I) can be part of this inadequate relation because the bachelor program followed by in-service teachers in the teacher training was shorten compared to the program for pre-service elementary teachers. So, the Institution (I) has an important role to develop in-service elementary teachers' knowledge on multiplication a fraction by a whole number.

Even if some researchers may argue that multiplication of a fraction by a fraction is more complicated than multiplication of a fraction by a whole number, it appears not always true for Indonesian in-service elementary teachers. They could perform better on such problems and tended to use more homogeneous technology (θ) such as measurement of area (B1). They did not use this technology (θ) to desing a contextul problem about multiplication of a fraction by a whole number. It is caused by difficulties of in-service elementary teachers to find the relation between multiplication of a fraction by a whole number and multiplication of a fraction by a fraction.

As conclusion, we can see the relation of subjects X in teacher education institution I to knowledge objects O. Only 22% of Indonesian in-service elementary teachers (X) demonstrated a satisfactory relation to O_1 while 64% did in the context of O_2. It means there is a need to develop strong praxeologies at I in order to impart knowledge about fractions to the Indonesian in-service elementary teachers (X).

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REFERENCES

Barbé, J., Bosch, M., Espinoza, L., & Gascón, J. (2005). Didactic restrictions on teachers practice – the case of limits of functions in Spanish high schools. *Educational Studies in Mathematics,* 59, 235-268.

Chevallard Y. (1992). Fundamental concepts in didactics: Perspectives provided by an anthropological approach. In: Douady R, Mercier A (eds) Research in Didactique of Mathematics. Selected Papers. La Pensée sauvage, Grenoble, France, pp 131-167.

Durand-Guerrier, V., Winsløw, C., & Yoshida, H. (2010). A model of mathematics teacher knowledge and a comparative study in Denmark, France and Japan. In *Annales de Didactique et des Sciences Cognitives*, *15*, 147-172.

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Isiksal, M., & Cakiroglu, E. (2011). The nature of prospective mathematics teachers' pedagogical content knowledge: the case of multiplication of fraction. *Journal Mathematics Teacher Education,* 14: 213 – 230.

Leung, I. K. C., & Carbone, R. E. (2013). Pre-service teachers' knowledge about fraction division reflected through problem posing. *The Mathematics Educator,* 14, 80 – 92.

Luo, F. (2009). Evaluating the effectiveness and insights of pre-service elementary teachers' abilities to construct word problems for fraction multiplication. *Journal of Mathematics Education,* 2(1), 83 – 98.

Ma, L. (1999). *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States*. Mahwah, NJ: Lawrence Erlbaum Associates.

Putra, Z. H. (2016). Pengetahuan mahasiswa pedidikan guru sekolah dasar dalam merepresentasikan operasi pecahan dengan model persegi panjang. *Jurnal Elemen*, *2*(1), 1–13.

Putra, Z. H., & Winsløw, C. (2018). Teachers' collective knowledge: The case of equivalent fractions. In Journal of Physics: Conference Series (Vol. 1088). https://doi.org/10.1088/1742-6596/1088/1/012003\

Putra, Z. H., & Witri, G. (2017). Anthropological theory of the didactic: A new research perspective on didactic mathematics in Indonesia, *Jurnal Pendidikan Guru, 2*(1), 221-227.

Tirosh, D. (2000). Enhancing prospective teacher' knowledge of children's conceptions: The case of division of fractions: *Journal for Research in Mathematics Education*, *31*(1), 5 – 25.