

## Asessing Hyphotetical Learning Trajectory of Mathematics Teachers

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### Abstract

The purpose of this study is to provide an alternative for teachers in preparing teaching materials adapted to track students' mathematics learning in the classroom. This study is a qualitative research with case study method with data collection is a triangulation technique that focuses on 13 people of 5th grade elementary school teacher. The results showed that Hyphotetical Learning Trajectory greatly assist teachers in planning their mathematics teachings, especially in developing teaching materials in a coherent, logical, contextual, and cutting-edge, so that the learning is given to more effectively and efficiently. It was also revealed that the practice of teaching mathematics teacher in the classroom along with lesson plans they have prepared beforehand. Based on the implementation of Hyphotetical Learning Trajectory on mathematics teachers, it can be concluded that the knowledge and skills of teachers in teaching mathematics were well developed; the materials presented are in-depth; it can identify students' learning; it can anticipate the strategy, representation, and misconceptions of students, so that it can provide an inclusive appropriate the learning needs of their students.

**Keywords:** *Hyphotetical Trajectory Learning, Teaching Mathematics*

### 1. INTRODUCTION

Mathematics is abstract, while elementary school students are still at the stage of concrete operations. It is necessary for learning to consider (1) the child's development and how children learn; (2) strengths, needs, and interests of each child; (3) The family and community culture of each child (Copple & Bredekamp, 2009). Learning also noticed how the students develop the knowledge, understanding and skills from time to time (CCSM, 2010) so that all students have the

opportunity and support they need to learn with depth and understanding of each.

To achieve this needs the support given math teacher, the tasks and the corresponding equipment, discussion groups and the language used to define and build ideas (Confrey, Maloney & Corley, 2014) that requires a deep understanding of the hypothetical learning trajectory (HLT) students. By understanding the teacher can design HLT mathematics learning objectives, development path and a set of

instructional activities (Simon Clements, Sarama, 2009).

There have been many studies that discuss the child's learning trajectory but in this article seeks to contribute to the discussion about the picture of the processes used to build the model teacher students' thinking so that students can learn how inclusive in accordance with the learning styles and learning needs of each. In particular, this research question is how to design a math teacher learning by using HLT?

## 2. METHOD

This research is qualitative research using the case study method. All data is displayed in the form of descriptive. The study involved 13 teachers from 7 Grade 5 elementary school located in West Bandung regency. The study begins by assigning topics to be discussed, that angle, then the making of instructional design angle, the discussion on the working group of teachers, a reflection on the students' work, and revision of instructional design and learning strategies.

Before, during, and after learning, all activity is logged and recorded teachers through interviews, classroom observations, video recordings, and document research. Studies carried documents to see preparations made by the teacher teaching, especially regarding learning hypothetical trajectory. Open interviews conducted to confirm hypothetical learning trajectories conducted by the teacher. Classroom observations conducted to observe the behavior of students based on learning

hypothetical trajectory carried out by the teacher. This is done to see the development of the quality of learning by hypothetical learning trajectory. The video recording was made during the learning takes place.

Results of interviews transcribed and analyzed to describe the activities carried out by teachers. The results of the observation sheets were analyzed to get a general idea of the interactions that occur in the classroom that can be used as material for improving the quality of learning. This is also done to confirm the answers to the interview conducted on the teachers.

## 3. RESULT AND DISCUSSION

### a. Hypothetical Learning Trajectory the Teachers on the Material Angle

HLT in this study, compiled by a group of primary school teachers together with a discussion, see the practice of learning, making decisions, and improve the design of learning through reflection. Reflection of the teachers consists of three stages, namely reflection for action, reflection in action, and reflection of the action (Suryadi, 2010). First, teachers determine what topics will be discussed, which was to determine the angle. This topic was chosen because there are many students who are struggling in this material. Second, create learning design angle which include: learning objectives, instructional materials, instructional media, the methods used, the learning steps up to the vote.

At this stage of reflection for action, each teacher preparing

instructional design with an analysis of last year's instructional design corner then led to a meeting of teachers. Based on the study of the documentation, the teacher immediately provide material corner angle by drawing on the blackboard. Based on interviews with teachers, students were passive during the teaching and learning results obtained are less satisfactory. Teachers then discuss and take decisions that instructional design should be modified to be more efficient and effective.

Taken together, the teacher makes learning trajectory angle and directed students to be able to mention the definition of angle, explaining various angles, determining the angle formed by two hands on the clock; great estimating angles; measuring

angles with a protractor; and draw angles using a protractor.

The angle is the corner (Depdiknas, 2001), the unit of measure is degrees. Great tool for measuring angles is called a protractor. Semicircular protractor, the amount of  $180^{\circ}$ . The angle is named according to its size, the magnitude right angle  $90^{\circ}$ ,  $180^{\circ}$  magnitude straightangle, and circle  $360^{\circ}$ . With reference to the corners, we can estimate the magnitude of a corner and gave his name. There angles less than  $90^{\circ}$  named the acute angle and the angle is more than  $90^{\circ}$  named an obtuse angle.

In determining the size of an angle, the teacher started learning by measuring the angle of a clock that is round. Consider the following picture.

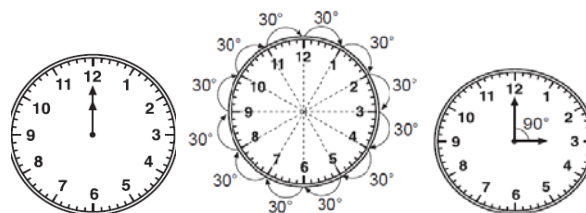


Figure 1. Determining the big corner using Watch

At 12 points, there are two needles, namely long and short needles. Minute's needle show while the long needle short needle show hours. The needle then moves to the right past the numbers 1, 2, 3, and so on until the back again point number 12. This means that the needle has spun a full turn or  $360^{\circ}$ . Therefore, the angles formed by the clock on any two numbers are the same distance, which is  $360^{\circ} : 12 = 30^{\circ}$ . For example, at 03.00 long needle pointing the number

12 and the short needle number 3. Large angle formed is  $30^{\circ} \times 3 = 90^{\circ}$ .

Furthermore, students are directed to be able to assess large an angle. Large angle of an object can be determined by estimating. Estimating generally closer the angle is obtained by measuring. Initially, the students estimate a great corner by comparing a corner with an angle of  $90^{\circ}$ . Then determine the amount of an angle with a large measure angles using non-standard measuring tool (eg using

paper shaped corner  $90^\circ$  further new measure angles using a protractor.

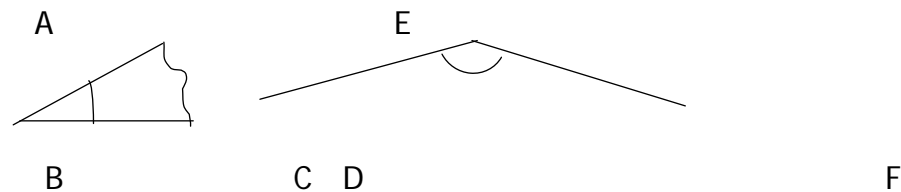


Figure 2. Determine and draw angles

Teachers then analyze student obstacles, predict the response and create anticipation didactic and pedagogic. Results of reflection for this action in the form of a didactical design hypothetic (DDH).

At this stage of reflection in action, teachers need to implement DDH by holding open class simulated by a model teacher, other teachers observe learning activities. In presenting the subject matter, the teacher gives a matchstick and students form various angles. Students can quickly mention the definition of angle, mentioning various forms of the angle, determines the angle formed by the two needle on the clock and can estimate the angle. In measuring angles with a protractor, students have difficulty using a protractor. The difficulties experienced by students are that they are not precise in determining the vertex and often reversed in a large reading angle on the protractor. Students guided to be able to measure angles using a protractor together with the group. Once the student is able to measure angles correctly, students directed to draw angle using a protractor. Teachers guide students who have difficulty with the questions that guide. At the end of the lesson, the

teacher rectifying students' understanding is wrong.

After the open class activities, teachers carry out reflection of the action. They analyzed the activity and student responses, learning path, learning media, collaboration and efficiency. Observations show that the students are actively involved during the lesson, the teacher models have been doing the learning according to the learning path that has been prepared, as well as instructional media used is also contextual. The results of the data obtained following the interview.

T : Teachers can direct students to be active in the learning process for all students holding a matchstick and form various angles. While there are student seen disturb other students but the teachers can lead students to be active learning.

Q : There are some changes in the implementation of learning for students adjusted to the conditions, but still in line with the planned learning path.

Q : The use of media has been effective, interest and contextual. Matchstick the teacher very easily found around the neighborhood students and cheap price,

Q : Some students did not cooperate but the teacher is able to direct the student to learn to share along with a group of their friends. Teachers give attention to the whole class so that all the groups were active learning.

T :Time used quite effectively and efficiently according to plan.

### **b. Reflections on Teacher's Hypothetical Learning Trajectory**

The good learning design is design that considers HLT student learning. Instructional design should consider the purpose of learning, learning activities, and ways of thinking and learning of students (Simon, 1995; Clements and Sarama, 2004; Battista, 2011) as well as observing how students develop the knowledge, understanding and skills from time to time (CCSM, 2010) the emphasis on mathematical concepts and reasoning (Confrey and Maloney, 2012).

HLT will help the teacher to look at a problem from the point of view of the students, it will help teachers to consider learning tasks to be provided and the appropriate equipment, discussion groups and the language used to define and build ideas (Confrey, Maloney & Corley, 2014). After consider the learning tasks are required, then the teacher can create a set of learning that has been adjusted to the level of thinking and development of learners. This task designed to help children learn new ideas and skills necessary to achieve the level of thought. Teaching materials arranged so that teachers will be coherent, ie ranging from simple to complex, from easy to hard and

concrete to the abstract in accordance with the purpose of learning. Breadth and depth of teaching materials can also be prepared by considering students' learning and students' potential obstacle, namely attention learners are learn sooner or later, students who have high and low motivation. Teachers also will design teaching materials appropriate to the context of student life and development of science and technology so that the source of learning and learning media used can varied.

HLT will guide the process of teaching (Van Den Heuvel-Panhuizen, 2001) in the direction of better and can lead to learning situations (Petit, 2011). HLT also can cause mental processes through the developmental levels of thinking (Graivemeijer, 2008). This allows teachers to achieve the learning objectives (Daro, Mosher, Corcoran, 2011).

By making HLT then the teacher's knowledge will evolve (Sztajn et al., 2012) by bridging research on teaching and research on teaching (Daro, Mosher, and Corcoran, 2011; NRC, 2007; Jacobs, Lamb & Philipp, 2010; Confrey et al., 2009). The development of teacher knowledge do teachers in professional development (Jacobs, Lamb & Philipp, 2010). Teachers can be the opportunity to collaborate (Wawro et al., 2012), so that their mutual sharing of knowledge, experience and teaching skills. Taking into account the HLT, then learning provided can more effective and efficient (Simon, 1995; Clements et al, 2011; Myers, 2014). Potential lesson planning could be improved (Graivemeijer, 2008; Wilson, Sztajn,

and Edgington, 2013; Morge et al., 2015), the material presented in-depth (Clements & Sarama, 2009), can anticipate the strategies of learners, representations, and misunderstandings (Maloney & Confrey, 2013, Wilson, 2009).

In this study, HLT do teachers together in a group of primary school teachers who interact with a discussion, practice learning, decision-making and improve the design of learning through reflection. Reflection involves taking past experience as a starting point (Dewey, 1933). Reflection done by teachers consists of three stages, namely: reflection for action, reflection in action and reflection of the action.

For reflection for action, teachers predict the flow of student learning by making teaching materials ranging from simple to complex, from the concrete to the abstract by developing guesses about student responses may occur over the situation didactic developed on purpose and teaching materials (Clements and Sarama, 2013; Clements, 2007), needs and potential of the students in learning how to create a situation didactic conducive, how to convey the lesson material that is supported by textbooks, objects manipulative or with the help of technologies (Jahnke, 2014; Kerres, 2003), a task that will be given, to how to design the assessment of learning and enrichment (Smart, 2011) and remedial for students. In addition, teachers also important to reflect on their learning difficulties or obstacles experienced by students. Based on these allegations, the teacher can develop anticipation anticipation include didactic and pedagogical. Results of

reflection for action is in the form of a didactical design hypothetic (DDH).

At this stage of reflection in action, teachers do open class by implementing DDH, teachers also reflection during the learning activities. Reflection done by developing millieu to createdidactic and pedagogical situations correspond to the needs of students. At this stage, the teacher HLT and anticipate applying student response, thus avoiding the possibility of Jourdain effect or Topaze effects. Teachers should also be able to develop course material is structurally and anticipate students' learning experience and knowledge (Suryadi, 2010). Teachers can use the questions that guide students in solving problems.

Teachers do not directly give the material a corner, but but invites students to identify known problems one by one, manipulate objects and provide questions that guide. In determining the amount of a corner, the teacher uses the angle formed by the hour so that students can easily calculate the corner. In search of great angles and draw angles using a protractor, the teacher taught first how to use a protractor to reduce mistakes made by the students.

In the reflection phase of action, the teacher models analyzing the activity of teachers and students, teaching materials and student learning outcomes shortly after the open class activities. Through this analysis, teachers can give students the widest possible opportunity to learn and implement the knowledge mastered. The results compared with the previously created instructional design and instructional design in order to

revise the design of learning can provide the widest possible space for students to have independent thinking through a design facility design didactic teacher.

The series of activities of reflection in action, and reflection of the action were filmed using video. Video recordings analyzed as consideration for effectiveness and teachers learning activities (Vrikki, 2017) and DDH revise instructional design created so that teachers can bring students to think critically so to solve the problems they face. Analysis of the video footage and activities of teachers in the teacher community greatly assist teachers in making learning HLT so the design can made according to the characteristics and learning needs of students. The results of the analysis in the form of instructional design called Didactical Design Empiric (Suryadi, 2015).

#### **4. CONCLUSION AND RECOMENDATION**

##### **a. Conclusion**

The activities of teachers in preparing teachers HLT communities, seen a change of mindset and attitudes of teachers. Competencies required of teachers also seen to increase, this can seen from the way teacher design learning and teaching. Student-centered learning approach with observe characteristics and learning needs of students through the steps of learning activities that are designed so as to provide ample opportunity for students to think and actualize himself. Selection of instructional media and learning resources be varied and the problems that made the problem

solving. This happens because the mutual sharing teachers with other teachers. Senior teachers can share experiences with junior teachers, junior teachers who can share their knowledge with the senior teacher. Instructional design which teachers become rich in anticipation of didactic and pedagogical so that confidence of teachers in developing instructional design and implement, increases. In addition, teachers who initially not directly involved in making teaching materials become increasingly active and able to work together to design instructional design to make HLT. They seemed enthusiast in every reflection, criticized the textbooks used in primary schools and create problems that lead to high-level thinking in grade 5. It is very important for teachers to be able to design appropriate learning to student characteristics, needs student learning, and can overcome learning difficulties experienced by students so as to make the design more effective learning and efficient. In particular, analysis and discussion of the teachers in the activities of preparing the design of learning by using HLT together with communities of teachers giving them the opportunity to develop knowledge and skills in the teaching of mathematics in order to develop properly, the material presented in-depth, can identify student learning, can anticipating strategies, representations, and misconceptions of students, so as to provide an inclusive services in accordance with the learning needs of their students.

## b. Recommendation

Based on the conclusions mentioned above, the researcher hopes and suggests: (1) Teachers can apply learning communities for design of learning with HLT because HLT make design effective dan efficient; (2) for Researchers by applying HLT to make used as a reference or basis to improve learning outcomes in mathematics or other subjects.

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